PRACTICE PROBLEMS 1 – Solutions to Additional Questions

#6/ (a) $p(w) \geq 0$, $p(1)+p(2)+p(3)+p(4)+p(5) = \frac{1}{55} + \cdots + \frac{25}{55} = 1$
so $p(w)$ is a probability function

(b) $p(w) \geq 0$, $\sum_{w=3}^{\infty} p(w) = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 \sum_{w=0}^{\infty} \left(\frac{2}{3}\right)^w = \frac{1}{3} \cdot \frac{8}{27} \cdot \frac{1}{1-\frac{2}{3}} = \frac{8}{27}$

so $\frac{27}{8} p(w)$ is a probability function.

(c) $p(w) \geq 0$, $\sum_w p(w) = 9$ so $\frac{1}{9} p(w)$ is a probability function.

(d) $p(w) \geq 0$, $\sum_w p(w) = \infty$ so $p(w)$ is not a probability function and there exists no a

(e) $p(w) \geq 0$, $\sum_{w=1}^{N} w = \frac{N(N+1)}{2}$ so $\frac{2}{N(N+1)}$ is a probability function

(f) $p(w) \geq 0$, $\sum_{w=1}^{\infty} \frac{1}{w} = \infty$ so $p(w)$ is not a probability function and there exists no a.

#7/ (a) $P(\{1,2,3\}) = e^3(1-e^{-3})^2\left[e^{-3} + 2e^{-6} + 3e^{-9}\right]$

(b) $P(\{2,3,4,\ldots\}) = e^3(1-e^{-3})^2 \sum_{k=2}^{\infty} k e^{-3k} = e^3(1-e^{-3})^2 e^{-3} \sum_{k=2}^{\infty} k (e^{-3})^{k-1}$

$$= (1-e^{-3})^2\left[\frac{1}{(1-e^{-3})^2} - 1\right]$$

(c) $\sum_{k=1}^{\infty} Cke^{-3k} = Ce^{-3} \sum_{k=1}^{\infty} k(e^{-3})^{k-1} = Ce^{-3} \frac{1}{(1-e^{-3})^2} = 1$

In both (b) and (c) I used the fact

$$\sum_{k=1}^{\infty} k x^{k-1} = \frac{d}{dx} \sum_{k=1}^{\infty} x^k = \frac{d}{dx}\left(\frac{1}{1-x}\right) \text{ for } |x|<1$$

$$= \frac{1}{(1-x)^2}$$

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