STA 257 – Fall 2002

Practice Problems 10

Recommended preparation for quiz to be held in tutorial on Wednesday, December 4

Material covered during the week of November 25: probability generating functions (3.10, 6.7), moment generating functions (3.9, 4.10, 5.6, 6.5), Central Limit Theorem (7.4)

Questions from the textbook (Schaeffer):

1. From Section 4.10: 4.95
2. From Chapter 5 Supplementary Exercises: 5.68
3. From Section 6.5: 6.15
4. From Section 7.4: 7.9, 7.17, 7.40

Additional questions:

5. Find EX and VX for the probability function from question 12 of Practice Problems 9 using the probability generating function. [Ans: $2q/p$, $2q/p^2$]

6. The function $\pi(t) = C(e^t + e^{-t})$ happens to be a probability generating function if the constant C is chosen correctly.

   (a) Find $P(X = 1)$ and $P(X = 10)$ in terms of C.

   (b) Find EX in terms of C.

   (c) Find C.

   [Ans: (a) 0, $2C/10!$ (b) $C(e^2 - 1)/e$ (c) $e/(e^2 + 1)$]

7. (a) Let X have the $N(\mu, \sigma^2)$ distribution. Find the moment generating function of X by using the fact that $X = \sigma Z + \mu$ where Z has the standard normal distribution. Then use the moment generating function to confirm that $EX = \mu$ and $VX = \sigma^2$.

   (b) Let $X_1, X_2, \ldots, X_n$ be i.i.d., each having the normal distribution with parameters μ and σ^2. Find the moment generating functions of the sample sum and the sample average. What are the distributions of these two random variables?

   [Ans: (a) $m_X(t) = e^{\mu t} e^{\sigma^2 t^2/2}$ (b) $m_{S_n}(t) = e^{n\mu t + n\sigma^2 t^2/2}$, $N(n\mu, n\sigma^2)$; $m_{\overline{X}_n}(t) = e^{\mu t + \sigma^2 t^2/2n}$, $N(\mu, \sigma^2/n)$]

8. Find the moment generating function of the Poisson distribution with parameter λ. Do it in two ways:

   (a) Use $m(t) = E(e^{tX}) = \sum_{k=0}^{\infty} e^{tk} \lambda^k e^{-\lambda}/k!$.

   (b) Use $m(t) = \pi(e^t)$ along with the formula for $\pi(t)$ found in question 10 of Practice Problems 9.

   [Ans: $m(t) = e^{\lambda(e^t - 1)}$]

9. At a certain bank, the deposits made by customers in the past have averaged \$638, with a standard deviation of \$126; but apart from this, nothing is known about the distribution of deposits. As part of a study, 75 deposits are chosen independently and at random, and they are found to average \$671. If the expected value and standard deviation have not changed, what is the approximate probability that the average of 75 independent deposits is \$671 or greater? Do you think it was reasonable to assume that the expected value and standard deviation have not changed? [Ans: 0.0116]

10. Let $X_1, X_2, \ldots, X_{50}$ be independent observations of some random variable X whose expected value is 100 and whose variance is 25. Let $\overline{X}_{50}$ be their average.

    (a) What are the expected value and variance of $\overline{X}_{50}$?

    (b) What does Chebyshev's inequality say about the probability that $\overline{X}_{50}$ fails to be between 98 and 102?

    (c) What does the Central Limit Theorem say about the probability described in part (b)?

    [Ans: (a) 100; 1/2 (b) It is less than or equal to 1/8 (c) It is approximately equal to 0.0046]

Material to be covered during the week of December 2: material on Statistics hand-out