STA 257 – Fall 2002

Practice Problems 1

Recommended preparation for quiz to be held in tutorial on Wednesday, September 18

Sections from Schaeffer covered in the week of September 9: all of Chapter 1, Sections 2.1, 2.3, 3.1, 3.3, 3.4, 3.5, 3.6, 3.7 (excluding expectation (E) and variance (V) in the last 5 sections). Other sections to review: Section 2.2 (Set Notation), Section 2.4 (Counting Rules).

Questions from the textbook (Schaeffer):

1. From Section 2.3: 2.5, 2.9
2. From Section 3.4: 3.25, 3.29
3. From Section 3.6: 3.43
4. From Section 3.7: 3.55
5. From Supplementary Exercises: 3.97, 3.109, 3.111, 3.113

Additional questions:

6. Which of the following are probability functions? For those that are not, find (if possible) a constant a so that $ap(\omega)$ is a probability function.

   (a) $p(\omega) = \frac{\omega^2}{55},\ \omega = 1, 2, 3, 4, 5$
   (b) $p(\omega) = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{\omega},\ \omega = 3, 4, 5, 6, \ldots$
   (c) $p(\omega) = 1$ for each ω in a nine-member set S
   (d) $p(\omega) = 1$ for each ω in a countably infinite set S
   (e) $p(\omega) = \omega,\ \omega = 1, 2, 3, 4, \ldots, N$
   (f) $p(\omega) = \frac{1}{\omega},\ \omega = 1, 2, 3, 4, \ldots$

   [Ans: (a) Yes, (b) $a = \frac{27}{8}$, (c) $a = \frac{1}{9}$, (d) Impossible, (e) $a = \frac{2}{N(N+1)}$, (f) Impossible]

7. Let probabilities of subsets of $S = \mathbb{N}$ be given by the probability function $p(k) = Cke^{-3k}$ $(k = 1, 2, 3, \ldots)$, where C is the constant $e^3(1 - e^{-3})^2$.

   (a) Find the probability of the event $\{1, 2, 3\}$.
   (b) Find the probability of the event $\{2, 3, 4, \ldots\}$.
   (c) Check that the probabilities add to 1. [*Hint:* Ignore the constant C for the moment and show that the terms ke^{-3k} add to its reciprocal. To do this, replace the 3 by an x and notice that the sum of ke^{-kx} is the derivative of another series (consider the Maclaurin series for $1/(1 - y)$) that you can evaluate as a geometric series. Differentiate the sum of that series and then put 3 back in for x.]

   [Ans: (a) 0.9995, (b) 0.0971]

Relevant sections in Schaeffer for lectures during the week of September 16: 4.1, 4.3, 4.4 (excluding expectation (E) and variance (V)).