STA 257S - Summer, 1996
Test #2
July 29, 1996

INSTRUCTIONS:

- Time: 50 minutes
- No aids allowed.
- Answers that are algebraic expressions should be simplified. Series and integrals should be evaluated. Numerical answers need not be expressed in decimal form.
- Total points: 35

NAME:_____

STUDENT NUMBER:_____

TUTOR:_____

1. (5 points) A student takes a multiple choice test. Each question has four possible answers. She knows the answers to 50% of the questions, can narrow the choice down to two answers 30% of the time, and does not know anything about the remaining 20% of the questions. What is the probability that she'll correctly answer a question chose at random from the test?

2. (5 points) A, B, and C are events with $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.5$, A and B are disjoint, A and C are independent, and $P(B|C) = 0.1$. Find $P(A \cup B \cup C)$.

3. Suppose the random variables X and Y have joint density function

$$f(x,y) = \begin{cases} \frac{6}{7}(x+y)^2 & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) (5 points) Find $P(X + Y \le 1)$.

(b) (5 points) Find the marginal density of Y.

(c) (3 points) Are X and Y independent? Explain.

4. (7 points) X_1, X_2, and X_3 are independent random variables with variances $\mathrm{Var}(X_1) = 1$, $\mathrm{Var}(X_2) = 4$, and $\mathrm{Var}(X_3) = 9$. Find the correlation between $Y = X_1 - X_2$ and $Z = X_2 + X_3$.

5. (5 points) A machine used to fill cereal boxes dispenses, on average, μ grams per box. The manufacturer wants the actual grams dispensed, X, to be within two grams of μ at least 75% of the time. What is the largest value of σ^2, the variance of X, that can be tolerated if the manufacturer's objectives are to be met?