UNIVERSITY OF TORONTO

Faculty of Arts and Science

AUGUST EXAMINATIONS 1996 STA 257S

Duration - 3 hours

NO AIDS ALLOWED

(Questions: 15;

Pages: 10;

Total Points: 115)

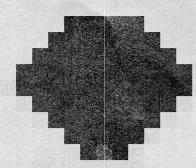
Answer all questions in the space provided. Show all of your work.

NAME:

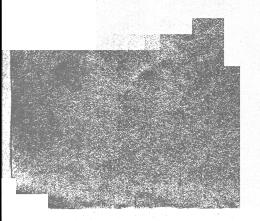
STUDENT NUMBER:

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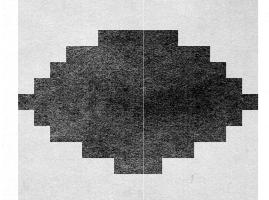
1. (5 points) Suppose there are three cabinets labelled A, B, and C, each of which has two drawers. Each drawer contains one coin. In cabinet A there are two gold coins, in cabinet B there are two silver coins, and in cabinet C there is one gold and one silver coin. A cabinet is chosen at random, one of the drawers is opened, and a silver coin is found. What is the probability that the other drawer in that cabinet contains a silver coin?



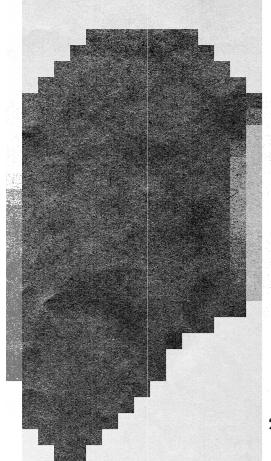
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- 2. (10 points) Prove each of the following.
 - (a) For any two events A and B, $P(A \cap B) \ge 1 P(A^c) P(B^c)$.



(b) If $P(B|A^c) = P(B|A)$, then A and B are independent.



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- 3. (15 points) Let X be a discrete random variable with probability mass function $P(X=n)=(1-\theta)\theta^{n-1}, \ n=1,2,3,\ldots,\ (0<\theta<1).$
 - (a) Show $P(X > n) = \theta^n$.

(b) Show P(X > m + n | X > m) = P(X > n).

(c) Let X_1 and X_2 be independent discrete random variables with probability mass functions

$$P(X_1 = n) = (1 - \theta_1)\theta_1^{n-1}, \ n = 1, 2, 3, \dots \ (0 < \theta_1 < 1)$$

$$P(X_2 = n) = (1 - \theta_2)\theta_2^{n-1}, \quad n = 1, 2, 3, \dots (0 < \theta_2 < 1)$$

Show $P(X_2 > X_1) = \frac{\theta_2(1-\theta_1)}{1-\theta_1\theta_2}$.

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4. (4 points) A random variable X' is said to be obtained from the random variable X by "truncation at the point a" if X' is defined by

$$X'(\omega) = \begin{cases} X(\omega) & \text{if } X(\omega) \le a \\ a & \text{if } X(\omega) > a \end{cases}$$

Express the distribution function of X' in terms of the distribution function of X.

5. (7 points) X and Y are jointly distributed discrete random variables with joint mass function given in the table:

Using the information that $P(Y=2|X=0)=\frac{1}{4}$ and that X and Y are independent, fill in the missing information in the table.

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- 6. (17 points) Suppose X and Y are uniformly distributed over the triangle with vertices (-1,0), (0,1), and (1,0).
 - (a) What is the joint density function of X and Y?

(b) Find $P(X \le \frac{3}{4}, Y \le \frac{3}{4})$.

(c) Find E(XY).

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7. (5 points) Suppose X and Y are discrete random variables with joint probability mass function p(x,y). Prove E(aX+bY)=aE(X)+bE(Y), where $a,b\in\mathbf{R}$.

8. (5 points) If $X \sim \text{Unif}(0,1)$ find the density of $Y = -2\log(X)$.

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- 9. (15 points) Suppose X and Y are independent, identically distributed exponential random variables with parameter λ .
 - (a) Find $P(X \ge Y \ge 2)$.

(b) Find the joint density function of $U = \frac{X}{Y}$ and V = X + Y. Are U and V independent?

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10. (3 points) Suppose T has a t distribution with n degrees of freedom. What is the distribution of T^2 ? State the value of any parameters.

11. (8 points) Use probability generating functions to find the distribution of $Y = X_1 + X_2$ where X_1 and X_2 are Poisson random variables with parameters λ_1 and λ_2 , respectively. (The probability mass function for the Poisson distribution is $p(k) = \frac{\lambda^k e^{-\lambda}}{k!}$, k = 0, 1, 2, ...)

12. (6 points) For each of the following functions, state whether or not it is a moment generating function. If not, explain why not. If so, find the underlying distribution.

(a)
$$m(t) = \frac{e^t}{4-e^t}$$

(b)
$$m(t) = \frac{3e^{4t} + e^{-2t}}{4}$$

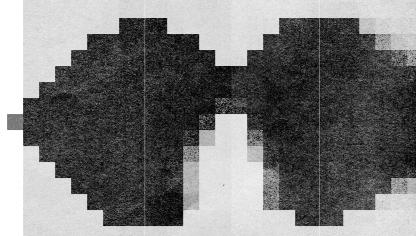
13. (5 points) Let m(t) be the moment generating function of the random variable X and define $\kappa(t) = \log m(t)$. Show that

$$\left. \frac{d}{dt} \kappa(t) \right|_{t=0} = E(X)$$

and

$$\left. \frac{d^2}{dt^2} \kappa(t) \right|_{t=0} = \mathrm{Var}(X).$$

14. (5 points) Let X be a non-negative random variable such that its moment generating function, m(t), is finite for all t. Prove $P(X \ge a) \le e^{-ta}m(t)$ for $t \ge 0$, and a a positive constant.



15. (5 points) A fair die is rolled 12000 times. Let S be the total number of sixes. Use the Central Limit Theorem to find P(1900 < S < 2200) in terms of Φ , the standard normal distribution function.

