In 1938, Duke University researchers Pratt and Woodruff conducted an experiment looking for evidence of ESP (extrasensory perception). In the experiment, students were presented with five standard ESP symbols (square, wavy lines, circle, star, cross). The experimenter shuffled a deck of ESP cards, each of which had one of the five symbols on it. The experimenter drew a card from this deck, looked at it, and concentrated on the symbol on the card. The student would then guess the symbol, perhaps by reading the experimenter's mind.

This experiment was repeated with 32 students for a total of 60,000 trials. The students were correct 12,489 times. If the students were selecting one of the five symbols at random, the probability of success would be $\boxed{p=0.2}$ and we would expect the students to be correct 12,000 times out of 60,000.

if guessing

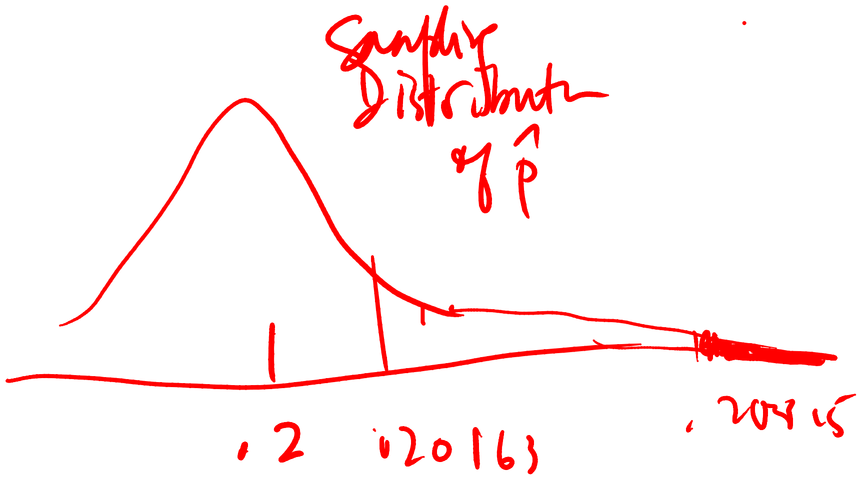
$\hat{p} = .20815 = \frac{12489}{60000}$

Should we write off the observed excess of 489 as nothing more than random variation?

Sampling distribution of $\hat{p} \sim N\left(p, \frac{p(1-p)}{n}\right)$

$n = 60000$

If ~~guessing~~ $\hat{p} \sim N\left(.2, \frac{.2(.8)}{60000}\right)$



$\cancel{P(.20815) = 0}$

$$s.d. = \sqrt{\frac{.2(.8)}{60000}} = .00163$$

If guessing
P(doing as well or better than they did)

$$\frac{.20815 - .2}{.00163} = 4.7 \text{ s.d above the mean}$$

Extremely unlikely

It seems the student might have ESP because if they were just guessing, it would be very unlikely to do as well as they did

99% CI for $\hat{p}$: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

$= .20815 \pm 2.576\sqrt{\dfrac{.20815(1-.20815)}{60000}}$

estimating p; don't assume that I know it

$= (.2039, .2124)$

Doesn't include .2, indicating they seem to be doing better than guessing

Conditions necessary for CI:

n large enough:

① $np \geq 10$, $n(1-p) \geq 10$

not a problem for $n = 60000$, $p = .2$

or $\hat{p} = .26815$

② All 60,000 observations should be independent.

But only on 32 students so maybe not.