STA 303 H1S / 1002 HS – Winter 2010
Test
February 25, 2010

LAST NAME: SOLUTIONS FIRST NAME:

STUDENT NUMBER:

ENROLLED IN: (circle one) STA 303 STA 1002

INSTRUCTIONS:

- Time: 90 minutes
- Aids allowed: calculator.
- Some formulae are on the last page (page 10).
- Total points: 45

1a	1bcd	2	3a	3b(i,ii,iii,iv)	3b(v,vi,vii)	4

1. A manufacturing facility needs to be able to switch from one type of package to another quickly to react to changes in orders. Consultants have developed a new method of changing the production line and used it to produce a sample of 48 change-over times (in minutes). Also available is an independent sample of 72 change-over times (in minutes) for the existing method. Does the mean change-over time differ between the two methods?

   Here is some output from SAS for these data.

```
                                 The GLM Procedure
                              Class Level Information

                          Class          Levels    Values
                          method              2    Existing New

                         Number of Observations Read         120
                         Number of Observations Used         120

Dependent Variable: changeover

                                         Sum of
Source                      DF          Squares    Mean Square   F Value   Pr > F

Model                        1       290.068056     290.068056      5.08   0.0260
Error                      118      6736.923611      57.092573
Corrected Total            119      7026.991667

            R-Square     Coeff Var      Root MSE    changeover Mean
            0.041279      45.54071      7.555963           16.59167

Source                      DF        Type I SS    Mean Square   F Value   Pr > F
method                       1      290.0680556    290.0680556      5.08   0.0260

Source                      DF      Type III SS    Mean Square   F Value   Pr > F
method                       1      290.0680556    290.0680556      5.08   0.0260

                                                       Standard
 Parameter                      Estimate                  Error    t Value    Pr > |t|

 Intercept                   14.68750000 B           1.09060928      13.47      <.0001
 method     Existing          3.17361111 B           1.40797053       2.25      0.0260
 method     New               0.00000000 B            .                .          .

NOTE: The X'X matrix has been found to be singular, and a generalized inverse
      was used to solve the normal equations.  Terms whose estimates are
      followed by the letter 'B' are not uniquely estimable.
```

   (a) (1 mark) Is there evidence of a difference in the means of change-over time between the two methods? Explain.

   Yes. We have moderate evidence ($p = 0.0260$) that the coefficient of the dummy variable that is 1 if the existing method is used is not 0.

(b) (2 marks) What are the means of the 48 change-over times from the new method and the 72 change-over times from the existing method?

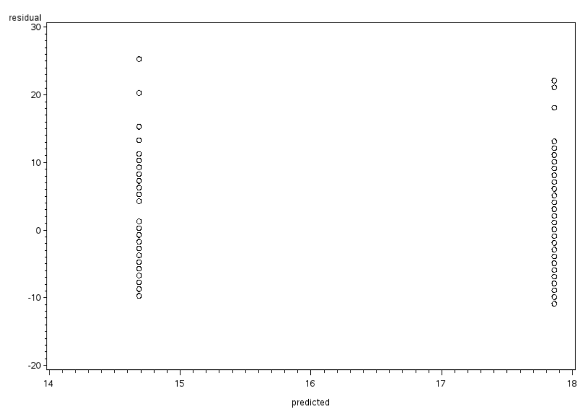
Existing method: mean is $14.69 + 3.17 = 17.86$
New method: mean is 14.69

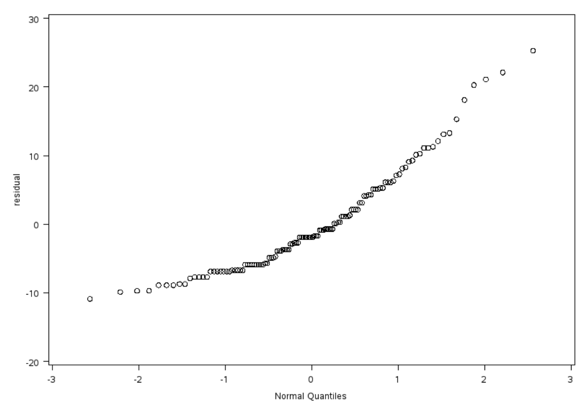
(c) (3 marks) Explain, in the context of this problem, the meaning of the following note produced by SAS:

```
NOTE: The X'X matrix has been found to be singular, and a generalized inverse
      was used to solve the normal equations.  Terms whose estimates are
      followed by the letter 'B' are not uniquely estimable.
```

There are two levels (new and existing) for the variable `method`. *SAS creates a dummy variable for both levels. The* $\mathbf{X}$ *matrix then has a column of 1's, a column that is 1 for the new method and 0 otherwise, and a column that is 1 for the existing method and 0 otherwise. Since every observation uses either the new or existing method, the sum of these second two columns gives a column of 1's, so the columns of* $\mathbf{X}$ *are linearly dependent. As a result* $\mathbf{X}'\mathbf{X}$ *is singular.*

(d) (3 marks) Below are a plot of the residuals versus the predicted values and a normal quantile plot of residuals. What do you conclude from them?





First plot: no outliers, variance of the the observations in the two groups appears to be approximately equal
Second plot: the model error terms are not normally distributed; the distribution is skewed

2. An alternative formulation of the model that could have been used in question 1 is

$$Y_{gi} = \theta_g + \epsilon_{gi}, \quad g = 1, 2$$

where Y_{gi} is the change-over time for the ith observation using the gth method and ϵ_{gi} are random errors. By the method of least squares, the estimates of θ_g are found by minimizing

$$\sum_{g=1}^{2} \sum_{i=1}^{n_g} (Y_{gi} - \theta_g)^2$$

with respect to θ_1, θ_2.

(a) (2 marks) Find the least squares estimates of θ_1 and θ_2.

Let S be the expression above that should be minimized.

$$\frac{\partial S}{\partial \theta_g} = -2 \sum_{i=1}^{n_g} (Y_{gi} - \theta_g)$$

Setting the above equal to 0 and solving gives

$$\hat{\theta}_g = \frac{\sum_{i=1}^{n_g} Y_{gi}}{n_g} = \overline{Y}_g$$

(b) (2 marks) How are θ_1 and θ_2 related to the parameters of the model fit in question 1?

The model fit in question 1 is

$$Y = \beta_0 + \beta_1 I_{existing} + \epsilon$$

where $I_{existing}$ is 1 if the existing method is used and 0 otherwise. Since the expectations of Y_{gi} should be the same for both models

$$\begin{aligned} \beta_0 + \beta_1 &= \theta_1 \\ \beta_0 &= \theta_2 \end{aligned}$$

3. A book on baseball uses regression analysis to compare the success of 30 Major League Baseball teams. One relationship the author considers is the linear relationship between market size (that is, the population, in millions, of the city associated with each team (variable name: `population`)) and the number of times the team made the playoffs in the 10 seasons between 1995 and 2004 (variable name: `appearances`). The author found that "it is hard to find much correlation between market size and success in making the playoffs. The relationship is quite weak."

   (a) (2 marks) The author's comments are about a linear regression analysis that was carried out. Indicate two concerns that potentially threaten the validity of this analysis.

   The number of playoff appearances is a count taking a value from $0, 1, 2, \ldots, 10$. *Thus it is not normally distributed. It could better be modeled as a binomial random variable. Then the variance is a function of the probability of making the playoffs, which will not be the same for all observations. So two of the assumptions of linear regression are violated.*

   (b) Some SAS output for an appropriate logistic regression analysis is given below and on the next page. A few numbers have been replaced by letters.

```
                    The LOGISTIC Procedure

                      Model Information
         Data Set                        WORK.A
         Response Variable (Events)      appearances
         Response Variable (Trials)      n
         Model                           binary logit
         Optimization Technique          Fisher's scoring

             Number of Observations Read          30
             Number of Observations Used          30
             Sum of Frequencies Read             300
             Sum of Frequencies Used             300

                       Response Profile
                Ordered     Binary           Total
                  Value     Outcome      Frequency
                      1     Event               80
                      2     Nonevent           220

                  Model Convergence Status
         Convergence criterion (GCONV=1E-8) satisfied.

              Deviance Goodness-of-Fit Statistic
  Criterion          Value        DF     Value/DF      Pr > ChiSq
  Deviance        116.2229       (A)       4.1508          <.0001

             Number of events/trials observations: 30
```

Model Fit Statistics

Criterion	Intercept Only	Intercept and Covariates
AIC	349.949	(B)
SC	353.653	351.483
-2 Log L	347.949	340.075

Testing Global Null Hypothesis: BETA=0

Test	Chi-Square	DF	Pr > ChiSq
Likelihood Ratio	(C)	1	0.0050

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	1	-1.4584	0.2110	47.7649	<.0001
population	1	0.0781	0.0275	8.0534	0.0045

Odds Ratio Estimates

Effect	Point Estimate	95% Wald Confidence Limits	
population	1.081	1.024	(D)

Obs	team	DevResid	Pearson Resid
1	Mets	-1.92105	-1.85370
2	Yankees	3.76061	3.20643
3	Angels	-1.22434	-1.16810
4	Dodgers	-0.52485	-0.51634
5	Cubs	-0.85685	-0.82115
6	WhiteSox	-1.65667	-1.49836
7	Phillies	-2.48767	-1.90432
8	Rangers	0.29713	0.30201
9	Marlins	-0.41610	-0.40514
10	Astros	1.68376	1.81046
11	BlueJays	-2.40481	-1.83112
12	Tigers	-2.38611	-1.81475
13	RedSox	1.72103	1.85669
14	Braves	5.30552	5.55467
15	Athletic	1.09465	1.15770
16	Giants	0.98205	1.02942
17	Expos	-2.30392	-1.74343
18	Diamondb	0.50449	0.52033
19	Mariners	1.21489	1.29822
20	Twins	0.53480	0.55290
21	Padres	-0.18949	-0.18692
22	Cardinal	1.91378	2.10315
23	Orioles	-0.16301	-0.16108
24	Pirates	-2.22632	-1.67701
25	DevilRay	-2.22363	-1.67472
26	Rockies	-0.97242	-0.89234
27	Indians	2.62450	2.95412
28	Reds	-0.95668	-0.87852
29	Royals	-2.18087	-1.63850
30	Brewers	-2.15560	-1.61721

i. (5 marks) Give the values of the missing numbers. ((D) is worth 2 marks.)

(A) = 28

(B) = 344.075

(C) = 7.874

(D) = 1.141

(D) = $\exp\{0.0781 + 1.96(0.0275)\}$

ii. (2 marks) Give the p-values for 2 tests with null hypothesis that the coefficient of population is 0.

0.0050 *and* 0.0045

iii. (2 marks) Explain what is being tested by the Deviance Goodness-of-Fit test.

The saturated model has as explanatory variables 29 indicator variables for the values of `population`*, i.e., it treats* `population` *as a categorical variable. The Deviance Goodness-of-Fit test has null hypothesis that the saturated model and fitted model fit the data equally well and alternative hypothesis that the saturated model fits the data better.*

iv. (2 marks) Explain in practical terms the interpretation of the estimated coefficient of `population`.

$e^{\hat{\beta}_1} = 1.081$
A 1 million increase in population increases the odds of making the playoffs by 8%.

v. (2 marks) What population is associated with an estimated 50% chance of making the playoffs?

This is the value of `population` *that gives a log of odds equal to 0. So the population with a 50% chance of making the playoffs is* $1.4584/0.0781 = 18.7$ *million.*

vi. (2 marks) What do you conclude from the residuals?

The model does not fit the data for the Braves which has a very large residual. The Braves made the playoffs much more than would be expected for the population of the city in which they play. (There are other observations with somewhat large (> 2 *in absolute value) residuals, but the number of playoff appearances for the Braves is extremely unusual.)*

vii. (4 marks) Does the fitted model appear to be appropriate from the SAS output you are given? What else would you like to see to assess the appropriateness of the model?

From what we are given there are problems with the model. There are outliers (as noted in `v.`*) and the deviance goodness-of-fit test gives strong evidence (*$p < 0.0001$*) that the saturated model is better than the fitted model.*
It would be useful to add polynomial terms in population to the model to see if they significantly improve the fit. Since this is a binomial response logistic regression, we could also look at a plot of the logit of the response proportions versus population to examine the nature of the relationship.

4. A textile researcher is interested in how four different colours of dye affect the durability of fabrics. Because the effects of the dye may be different for different types of cloth, he applies each dye to five different kinds of cloth. For each kind of cloth, 24 fabric specimens are cut from a length of the cloth and the first six of the 24 specimens are dyed the first colour, the second six the second colour, etc. All 120 specimens are tested for durability, measured as the length of time for the fabric to break down under a stress.

   Explain how you would carry out the analysis on the resulting data. In particular, indicate:

   (a) (1 mark) The type of analysis (one-way analysis of variance, two-way analysis of variance, binary response logistic regression, or binomial response logistic regression) to be carried out.
   two-way analysis of variance

   (b) (3 marks) The response variable and the explanatory variables as they will be entered into the model.

   Response variable: durability
   Explanatory variables: 3 indicator variables for colour of dye, 4 indicator variables for type of cloth, 12 interaction terms that are the products of the pairs of the indicator variables for colour and cloth

   (c) (5 marks) The test(s) you would carry out to evaluate effects of dye on the durability of the fabrics. For the test(s) indicate the null and alternative hypotheses and the probability distribution(s) (including the degrees of freedom) of the test statistic(s) under the null hypothesis.

   First test to see if the interaction is significant. The null hypothesis is that all of the 12 coefficients of the interaction terms are 0 and the alternative is that at least one of these coefficients is not 0. Under the null hypothesis, the test statistic has an F-distribution with 12 and 100 degrees of freedom (denominator degrees of freedom is the error degrees of freedom = 119-3-4-12).
   If the interaction is significant, stop (conclusions about the effects of the colour of the dye must be described differently for the different types of cloth).
   If the interaction is not significant, remove the interaction terms from the model and test for the main effect of colour. The null hypothesis will be that all 3 coefficients of the indicator variables for colour are 0 and the alternative hypothesis will be that at least one of these coefficients is not 0. Under the null hypothesis, the test statistic will have an F distribution with 3 and 112 degrees of freedom.

   (d) (2 marks) Do you have any concerns about the validity of the tests? Why or why not?

   It may not be reasonable to treat the observations as independent since they are taken on adjacent pieces of cloth. The lack of independence means the error estimate and the F tests are not valid.

Some formulae:

Pooled t-test

$$t_{obs} = \frac{\overline{y}_1 - \overline{y}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

Linear Regression

$$b_1 = \frac{\sum(X_i-\overline{X})(Y_i-\overline{Y})}{\sum(X_i-\overline{X})^2} = \frac{\sum X_iY_i - n\overline{XY}}{\sum X_i^2 - n\overline{X}^2} \qquad b_0 = \overline{Y} - b_1\overline{X}$$

One-way analysis of variance

$$\text{SSTO} = \sum_{i=1}^{N}(Y_i - \overline{Y})^2 \qquad \text{SSE} = \sum_{g=1}^{G}\sum_{(g)}(Y_i - \overline{Y}_g)^2$$

$$\text{SSR} = \sum_{g=1}^{G} n_g(\overline{Y}_g - \overline{Y})^2$$

Bernoulli and Binomial distributions

If $Y \sim \text{Bernoulli}(\pi)$
$\text{E}(Y) = \pi$, $\text{Var}(Y) = \pi(1-\pi)$

If $Y \sim \text{Binomial}(m, \pi)$
$\text{E}(Y) = m\pi$, $\text{Var}(Y) = m\pi(1-\pi)$

Logistic Regression with Binomial Response formulae

$$\text{Deviance} = 2\sum_{i=1}^{n}\{y_i\log(y_i) + (m_i - y_i)\log(m_i - y_1) - y_i\log(\hat{y}_i) + (m_i - y_i)\log(m_i - \hat{y}_1)\}$$

$$D_{res,i} = \text{sign}(y_i - m_i\hat{\pi}_i)\sqrt{2\left\{y_i\log\left(\frac{y_i}{m_i\hat{\pi}_i}\right) + (m_i - y_i)\log\left(\frac{m_i - y_i}{m_i - m_i\hat{\pi}_i}\right)\right\}}$$

$$P_{res,i} = \frac{y_i - m_i\hat{\pi}_i}{\sqrt{m_i\hat{\pi}_i(1-\hat{\pi}_i)}}$$

Model Fitting Criteria

$$\text{AIC} = -2\log(L) + 2(k+1) \qquad \text{SC} = -2\log(L) + (k+1)\log(N)$$