UNIVERSITY OF TORONTO

Faculty of Arts and Science

APRIL 2010 EXAMINATIONS
STA 303 H1S / STA 1002 HS

Duration - 3 hours

Aids Allowed: Calculator

LAST NAME:_____**FIRST NAME:**_____

STUDENT NUMBER: _____

• There are 27 pages including this page.
• The last page is a table of formulae that may be useful. For all questions you can assume that the results on the formula page are known.
• A table of the chi-square distribution can be found on page 26.
• Total marks: 90

1abcd	1efg	1hi	2ab	2cde	2fghi

3a	3bcdef	4abcd	4efg	5

1. A study was carried out to investigate the effects of heredity and environment on intelligence. From adoption registers, researchers selected samples of adopted children whose biological parents and adoptive parents came from either the very highest or the very lowest socio-economic status (SES) categories. They attempted to obtain samples of size 10 from each combination (1. high adoptive SES and high biological SES, 2. high adoptive SES and low biological SES, 3. low adoptive SES and high biological SES, and 4. low SES for both parents). However, only 8 children belonged to combination 3. The 38 selected children were given intelligence quotient (IQ) tests. Some output from SAS for this analysis is given below and on the next 2 pages. The variables `adoptive` and `biologic` each take on the values `High` and `Low`, indicating the SES of the respective parents.

		IQ		
		Mean	Std	N
adoptive	biologic			
High	High	119.60	12.25	10.00
	Low	103.60	12.71	10.00
Low	High	107.50	11.94	8.00
	Low	92.40	15.41	10.00

The GLM Procedure

Class Level Information

Class	Levels	Values
adoptive	2	High Low
biologic	2	High Low

Number of Observations Read 38
Number of Observations Used 38

Dependent Variable: IQ

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	(A)	(C)	1257.003509	7.19	0.0007
Error	(B)	5941.200000	(D)		
Corrected Total	37	9712.210526			

R-Square	Coeff Var	Root MSE	IQ Mean
0.388275	12.50799	13.21897	105.6842

Output continues on the next page

(Question 1 continued)

Source	DF	Type I SS	Mean Square	F Value	Pr > F
adoptive	1	1477.632749	1477.632749	8.46	0.0064
biologic	1	2291.471895	2291.471895	13.11	0.0009
adoptive*biologic	1	1.905882	1.905882	0.01	0.9174

Source	DF	Type III SS	Mean Square	F Value	Pr > F
adoptive	1	1277.388235	1277.388235	7.31	0.0106
biologic	1	2275.788235	2275.788235	13.02	0.0010
adoptive*biologic	1	1.905882	1.905882	0.01	0.9174

The GLM Procedure

Class Level Information

Class	Levels	Values
adoptive	2	High Low
biologic	2	High Low

Number of Observations Read	38
Number of Observations Used	38

Dependent Variable: IQ

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	3769.104644	1884.552322	11.10	0.0002
Error	35	5943.105882	169.803025		
Corrected Total	37	9712.210526			

R-Square	Coeff Var	Root MSE	IQ Mean
0.388079	12.32999	13.03085	105.6842

Source	DF	Type I SS	Mean Square	F Value	Pr > F
adoptive	1	1477.632749	1477.632749	8.70	0.0056
biologic	1	2291.471895	2291.471895	13.49	0.0008

Source	DF	Type III SS	Mean Square	F Value	Pr > F
adoptive	1	1276.005229	1276.005229	7.51	0.0096
biologic	1	2291.471895	2291.471895	13.49	0.0008

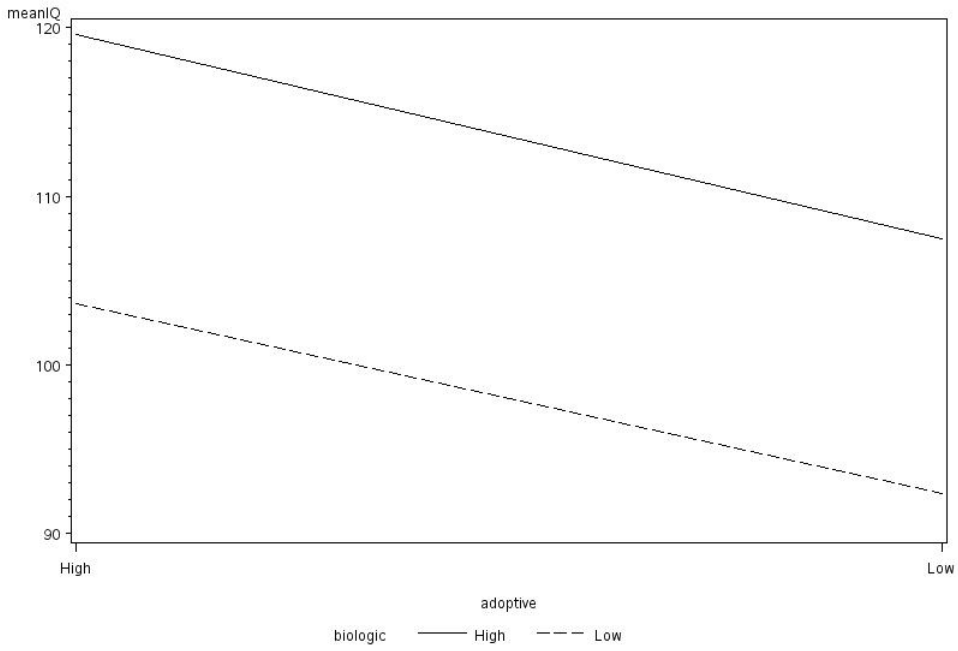
Level of adoptive	N	IQ Mean	IQ Std Dev
High	20	111.600000	14.6625193
Low	18	99.111111	15.6238464

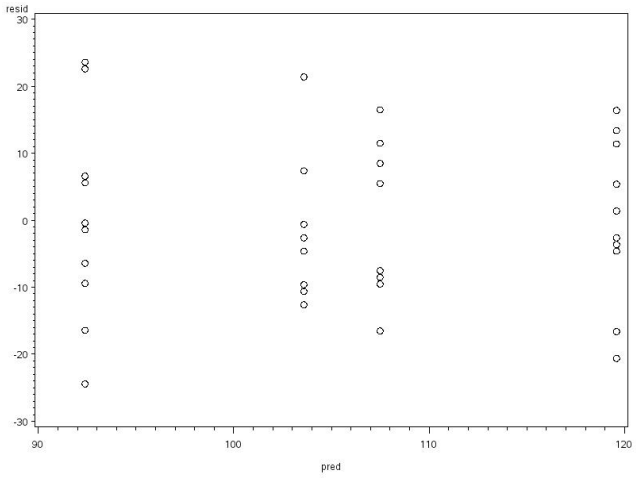
Least Squares Means

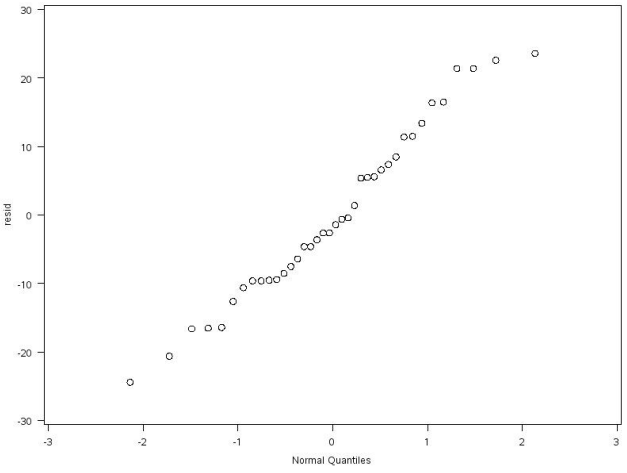
adoptive	IQ LSMEAN
High	111.600000
Low	99.976471

Output continues on the next page

(Question 1 continued)







Questions begin on the next page.

(a) (4 marks) Some numbers in the SAS output on page 2 have been replaced by letters. What are the missing values?

(A) = _____

(B) = _____

(C) = _____

(D) = _____

(b) (1 mark) Two linear models have been fit in the output above. In the first linear model, how many β's (coefficients of terms in the linear model) must be estimated?

(c) (2 marks) Why can the first model be considered a saturated model? Explain why, in this case, it is possible to carry out inference.

(d) (2 marks) What is being tested by the test with p-value 0.9174? What do you conclude?

(e) (2 marks) For the second linear model, some "Least Squares Means" are given. Explain clearly how they are calculated.

(f) (2 marks) Why does one of the "Least Squares Means" differ from the means given in the table above the least squares means?

(g) (3 marks) From the results of this study, what do you conclude about the relationship between parental socio-economic status and IQ? Quote relevant p-values to support your conclusions.

(h) (3 marks) The first graph on page 4 is a plot of the mean IQ of the children, classified by the socio-economic status of their adoptive and biological parents. Explain how it illustrates your conclusions from part (g).

(i) (4 marks) Do you trust your conclusions from part (g)? Why or why not?

2. Some of the debate about capital punishment in the U.S. has revolved around the rôle race plays in the decision to use it. The 674 subjects considered in this question were the defendants in murder cases in Florida between 1976 and 1987. SAS output for 4 models is given below and on the next 3 pages. The variables are:
`V` - the race of the victim (either black (`B`) or white (`W`))
`D` - the race of the defendant (either black (`B`) or white (`W`))
`C` - verdict for capital punishment (yes (`Y`) or no (`N`))

MODEL 1

The GENMOD Procedure

Model Information

Distribution	Poisson
Link Function	Log
Dependent Variable	count

Number of Observations Read	8
Number of Observations Used	8

Class Level Information

Class	Levels	Values
V	2	B W
D	2	B W
C	2	N Y

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	4	402.8353	100.7088
Scaled Deviance	4	402.8353	100.7088
Pearson Chi-Square	4	419.5584	104.8896
Scaled Pearson X2	4	419.5584	104.8896
Log Likelihood		2725.4956	
Full Log Likelihood		-220.4376	
AIC (smaller is better)		448.8752	
AICC (smaller is better)		462.2085	
BIC (smaller is better)		449.1930	

Algorithm converged.

Analysis Of Maximum Likelihood Parameter Estimates

Parameter		DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept		1	3.6172	0.1255	3.3713	3.8632	830.72	<.0001
V	B	1	-1.1753	0.0907	-1.3531	-0.9974	167.81	<.0001
V	W	0	0.0000	0.0000	0.0000	0.0000	.	.
D	B	1	-0.9277	0.0855	-1.0953	-0.7602	117.81	<.0001
D	W	0	0.0000	0.0000	0.0000	0.0000	.	.
C	N	1	2.1874	0.1279	1.9367	2.4380	292.53	<.0001
C	Y	0	0.0000	0.0000	0.0000	0.0000	.	.
Scale		0	1.0000	0.0000	1.0000	1.0000		

Continued

MODEL 2

The GENMOD Procedure

Model Information

Distribution	Poisson
Link Function	Log
Dependent Variable	count

Number of Observations Read	8
Number of Observations Used	8

Class Level Information

Class	Levels	Values
V	2	B W
D	2	B W
C	2	N Y

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	3	22.2659	7.4220
Scaled Deviance	3	22.2659	7.4220
Pearson Chi-Square	3	19.7018	6.5673
Scaled Pearson X2	3	19.7018	6.5673
Log Likelihood		2915.7803	
Full Log Likelihood		-30.1529	
AIC (smaller is better)		70.3058	
AICC (smaller is better)		100.3058	
BIC (smaller is better)		70.7030	

Algorithm converged.

Analysis Of Maximum Likelihood Parameter Estimates

Parameter			DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept			1	3.8526	0.1239	3.6097	4.0955	966.09	<.0001
V	B		1	-3.3737	0.2542	-3.8721	-2.8754	176.08	<.0001
V	W		0	0.0000	0.0000	0.0000	0.0000	.	.
D	B		1	-2.2751	0.1516	-2.5722	-1.9780	225.30	<.0001
D	W		0	0.0000	0.0000	0.0000	0.0000	.	.
C	N		1	2.1874	0.1279	1.9367	2.4380	292.53	<.0001
C	Y		0	0.0000	0.0000	0.0000	0.0000	.	.
V*D	B	B	1	4.4654	0.3041	3.8694	5.0614	215.64	<.0001
V*D	B	W	0	0.0000	0.0000	0.0000	0.0000	.	.
V*D	W	B	0	0.0000	0.0000	0.0000	0.0000	.	.
V*D	W	W	0	0.0000	0.0000	0.0000	0.0000	.	.

Continued

MODEL 3

The GENMOD Procedure

Model Information

Distribution	Poisson
Link Function	Log
Dependent Variable	count

Number of Observations Read	8
Number of Observations Used	8

Class Level Information

Class	Levels	Values
V	2	B W
D	2	B W
C	2	N Y

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	2	5.3940	2.6970
Scaled Deviance	2	5.3940	2.6970
Pearson Chi-Square	2	5.8109	2.9054
Scaled Pearson X2	2	5.8109	2.9054
Log Likelihood		2924.2162	
Full Log Likelihood		-21.7170	
AIC (smaller is better)		55.4339	
AICC (smaller is better)		139.4339	
BIC (smaller is better)		55.9106	

Algorithm converged.

Analysis Of Maximum Likelihood Parameter Estimates

Parameter			DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept			1	4.0610	0.1258	3.8145	4.3076	1042.18	<.0001
V	B		1	-4.9710	0.5675	-6.0833	-3.8588	76.74	<.0001
V	W		0	0.0000	0.0000	0.0000	0.0000	.	.
D	B		1	-2.2751	0.1516	-2.5722	-1.9780	225.30	<.0001
D	W		0	0.0000	0.0000	0.0000	0.0000	.	.
C	N		1	1.9526	0.1336	1.6908	2.2144	213.68	<.0001
C	Y		0	0.0000	0.0000	0.0000	0.0000	.	.
V*C	B	N	1	1.7045	0.5237	0.6780	2.7310	10.59	0.0011
V*C	B	Y	0	0.0000	0.0000	0.0000	0.0000	.	.
V*C	W	N	0	0.0000	0.0000	0.0000	0.0000	.	.
V*C	W	Y	0	0.0000	0.0000	0.0000	0.0000	.	.
V*D	B	B	1	4.4654	0.3041	3.8694	5.0614	215.64	<.0001
V*D	B	W	0	0.0000	0.0000	0.0000	0.0000	.	.
V*D	W	B	0	0.0000	0.0000	0.0000	0.0000	.	.
V*D	W	W	0	0.0000	0.0000	0.0000	0.0000	.	.
Scale			0	1.0000	0.0000	1.0000	1.0000		

Continued

MODEL 4

The GENMOD Procedure

Model Information

Distribution	Poisson
Link Function	Log
Dependent Variable	count
Number of Observations Read	8
Number of Observations Used	8

Class Level Information

Class	Levels	Values
V	2	B W
D	2	B W
C	2	N Y

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	1	0.3798	0.3798
Scaled Deviance	1	0.3798	0.3798
Pearson Chi-Square	1	0.1978	0.1978
Scaled Pearson X2	1	0.1978	0.1978
Log Likelihood		2926.7234	
Full Log Likelihood		-19.2098	
AIC (smaller is better)		52.4197	
AICC (smaller is better)		.	
BIC (smaller is better)		52.9758	

Algorithm converged.

Analysis Of Maximum Likelihood Parameter Estimates

Parameter			DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept			1	3.9668	0.1374	3.6976	4.2361	833.78	<.0001
V	B		1	-5.6696	0.6459	-6.9355	-4.4037	77.06	<.0001
V	W		0	0.0000	0.0000	0.0000	0.0000	.	.
D	B		1	-1.5525	0.3262	-2.1918	-0.9132	22.66	<.0001
D	W		0	0.0000	0.0000	0.0000	0.0000	.	.
C	N		1	2.0595	0.1458	1.7736	2.3453	199.40	<.0001
C	Y		0	0.0000	0.0000	0.0000	0.0000	.	.
V*D	B	B	1	4.5950	0.3135	3.9805	5.2095	214.78	<.0001
V*D	B	W	0	0.0000	0.0000	0.0000	0.0000	.	.
V*D	W	B	0	0.0000	0.0000	0.0000	0.0000	.	.
V*D	W	W	0	0.0000	0.0000	0.0000	0.0000	.	.
D*C	B	N	1	-0.8678	0.3671	-1.5872	-0.1483	5.59	0.0181
D*C	B	Y	0	0.0000	0.0000	0.0000	0.0000	.	.
D*C	W	N	0	0.0000	0.0000	0.0000	0.0000	.	.
D*C	W	Y	0	0.0000	0.0000	0.0000	0.0000	.	.
V*C	B	N	1	2.4044	0.6006	1.2273	3.5816	16.03	<.0001
V*C	B	Y	0	0.0000	0.0000	0.0000	0.0000	.	.
V*C	W	N	0	0.0000	0.0000	0.0000	0.0000	.	.
V*C	W	Y	0	0.0000	0.0000	0.0000	0.0000	.	.

Continued

(a) (4 marks) For each of the 4 models for which output is given, give a practical interpretation of the relationships among the variables (assuming that the model is appropriate).

(b) (4 marks) Show how the value for the "Full Log Likelihood" is calculated for model 1. Give your answer in terms of the observed counts y_{ijk}.

(c) (1 mark) For model 1, explain why the degrees of freedom for the "Criteria For Assessing Goodness Of Fit" is 4.

(d) (5 marks) Use a likelihood ratio test to compare the fits of models 1 and 3. State the null and alternative hypotheses, the test statistic, the distribution of the test statistic under the null hypothesis, the p-value, and your conclusion.

(e) (4 marks) Carry out the Deviance Goodness-of-Fit test for model 3. State the null and alternative hypotheses, the test statistic, the distribution of the test statistic under the null hypothesis, the p-value, and your conclusion.

(f) (2 marks) Using model 4, what is the estimated count of the number of cases with a verdict of capital punishment for which the defendant and victim were both white?

(g) (3 marks) Using model 4, estimate the odds of receiving a verdict in favour of capital punishment if the defendant was black.

(h) (4 marks) For model 4, what evidence is available from the SAS output that the model is adequate? What else would you like to see to ensure that the Wald tests are appropriate?

(i) (2 marks) Which of the 4 models would you choose for these data? Why?

3. Below is some additional output analysing the data from question 2. `nCapital` is the number of cases for which the verdict was for capital punishment.

MODEL A

The LOGISTIC Procedure

Model Information

Response Variable (Events)	nCapital
Response Variable (Trials)	m
Model	binary logit
Optimization Technique	Fisher's scoring

Number of Observations Read	4
Number of Observations Used	4
Sum of Frequencies Read	674
Sum of Frequencies Used	674

Response Profile

Ordered Value	Binary Outcome	Total Frequency
1	Event	68
2	Nonevent	606

Class Level Information

Class	Value	Design Variables
V	B	1
	W	0
D	B	1
	W	0

Model Convergence Status

Quasi-complete separation of data points detected.

WARNING: The maximum likelihood estimate may not exist.

WARNING: The LOGISTIC procedure continues in spite of the above warning. Results shown are based on the last maximum likelihood iteration. Validity of the model fit is questionable.

Model Fit Statistics

Criterion	Intercept Only	Intercept and Covariates
AIC	442.843	426.577
SC	447.356	444.630
-2 Log L	440.843	418.577

Testing Global Null Hypothesis: BETA=0

Test	Chi-Square	DF	Pr > ChiSq
Likelihood Ratio	22.2659	3	<.0001
Score	19.7018	3	0.0002
Wald	14.6545	3	0.0021

Output for MODEL A continues on the next page.

(Question 3 continued)

Output for MODEL A continued

Type 3 Analysis of Effects

Effect	DF	Wald Chi-Square	Pr > ChiSq
V	1	0.0032	0.9547
D	1	5.0991	0.0239
V*D	1	0.0020	0.9640

Analysis of Maximum Likelihood Estimates

Parameter		DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept		1	-2.0556	0.1459	198.5297	<.0001
V	B	1	-11.3015	198.8	0.0032	0.9547
D	B	1	0.8426	0.3731	5.0991	0.0239
V*D	B B	1	8.9663	198.8	0.0020	0.9640

Association of Predicted Probabilities and Observed Responses

Percent Concordant	35.3	Somers' D	0.261
Percent Discordant	9.1	Gamma	0.589
Percent Tied	55.6	Tau-a	0.047
Pairs	41208	c	0.631

MODEL B

The LOGISTIC Procedure

Model Information

(SOME OUTPUT OMITTED HERE THAT IS THE SAME AS FOR MODEL A)

Model Convergence Status

Convergence criterion (GCONV=1E-8) satisfied.

Model Fit Statistics

Criterion	Intercept Only	Intercept and Covariates
AIC	442.843	424.957
SC	447.356	438.496
-2 Log L	440.843	418.957

Testing Global Null Hypothesis: BETA=0

Test	Chi-Square	DF	Pr > ChiSq
Likelihood Ratio	21.8861	2	<.0001
Score	18.7847	2	<.0001
Wald	16.2460	2	0.0003

Output for MODEL B continues on the next page.

Continued

Output for MODEL B continued

Type 3 Analysis of Effects

Effect	DF	Wald Chi-Square	Pr > ChiSq
V	1	16.0262	<.0001
D	1	5.5889	0.0181

Analysis of Maximum Likelihood Estimates

Parameter		DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept		1	-2.0595	0.1458	199.3973	<.0001
V	B	1	-2.4044	0.6006	16.0262	<.0001
D	B	1	0.8678	0.3671	5.5889	0.0181

Odds Ratio Estimates

Effect	Point Estimate	95% Wald Confidence Limits	
V B vs W	0.090	0.028	0.293
D B vs W	2.382	1.160	4.890

Association of Predicted Probabilities and Observed Responses

Percent Concordant	35.3	Somers' D	0.261
Percent Discordant	9.1	Gamma	0.589
Percent Tied	55.6	Tau-a	0.047
Pairs	41208	c	0.631

(a) (4 marks) Give test statistics and p-values for **two** tests comparing models A and B. What do you conclude? (As part of your conclusion, you should be choosing one of model A or B.)

(b) (2 marks) For the model you chose in part (a), describe the relationship among the 3 variables.

(c) (2 marks) Using model B, estimate the odds of receiving a verdict in favour of capital punishment if the defendant and victim were both black.

(d) (2 marks) The SAS output for model A includes the message below. Explain what the message indicates.

```
                    Quasi-complete separation of data points detected.
WARNING: The maximum likelihood estimate may not exist.
WARNING: The LOGISTIC procedure continues in spite of the above warning. Results shown
         are based on the last maximum likelihood iteration. Validity of the model fit is
         questionable.
```

(e) (2 marks) For model A, what are the hypotheses for the likelihood ratio test under the heading "Testing Global Null Hypothesis: BETA=0" in the SAS output? What do you conclude?

(f) (2 marks) Do you prefer the analysis carried out on these data in question 2 or question 3? Why?

4. A study followed the orthodontic growth of 27 children (16 males and 11 females). At ages 8, 10, 12, and 14, the distance (in millimeters) from the center of the pituitary to pterygomaxillary fissure was measured. The investigators were interested in how the growth of this distance varied as the boys and girls grew. In the analysis below, age was treated as a categorical variable.

   Some SAS output is given below for 3 models that were fit to the resulting data.

MODEL I

The Mixed Procedure

Model Information

Dependent Variable	distance
Covariance Structure	Compound Symmetry
Subject Effect	subject(sex)
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

Class Level Information

Class	Levels	Values
sex	2	Female Male
subject	27	F01 F02 F03 F04 F05 F06 F07 F08 F09 F10 F11 M01 M02 M03 M04 M05 M06 M07 M08 M09 M10 M11 M12 M13 M14 M15 M16
age	4	8 10 12 14

Dimensions

Covariance Parameters	2
Columns in X	15
Columns in Z	0
Subjects	27
Max Obs Per Subject	4

Number of Observations

Number of Observations Read	108
Number of Observations Used	108
Number of Observations Not Used	0

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	470.49084642	
1	1	423.40853283	0.00000000

Convergence criteria met.

Output for MODEL I continues on the next page.

Output for MODEL I continued

Estimated R Correlation Matrix
for subject(sex) F01 Female

Row	Col1	Col2	Col3	Col4
1	1.0000	0.6245	0.6245	0.6245
2	0.6245	1.0000	0.6245	0.6245
3	0.6245	0.6245	1.0000	0.6245
4	0.6245	0.6245	0.6245	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
CS	subject(sex)	3.2854
Residual		1.9750

Fit Statistics

-2 Res Log Likelihood	423.4
AIC (smaller is better)	427.4
AICC (smaller is better)	427.5
BIC (smaller is better)	430.0

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	47.08	<.0001

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
age	3	75	35.35	<.0001
sex	1	25	9.29	0.0054
age*sex	3	75	2.36	0.0781

Continued

MODEL II

(The output was edited to remove Class Level Information and Number of Observations (both same as model I) and Iteration History (convergence criteria were met).)

The Mixed Procedure

Model Information

Dependent Variable	distance
Covariance Structures	Variance Components, Autoregressive
Subject Effect	subject(sex)
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Containment

Dimensions

Covariance Parameters	3
Columns in X	15
Columns in Z	27
Subjects	1
Max Obs Per Subject	108

Estimated R Correlation Matrix for subject(sex) F01 Female

Row	Col1	Col2	Col3	Col4
1	1.0000	-0.05822	0.003390	-0.00020
2	-0.05822	1.0000	-0.05822	0.003390
3	0.003390	-0.05822	1.0000	-0.05822
4	-0.00020	0.003390	-0.05822	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
subject(sex)		3.3423
AR(1)	subject(sex)	-0.05822
Residual		1.9206

Fit Statistics

-2 Res Log Likelihood	423.3
AIC (smaller is better)	429.3
AICC (smaller is better)	429.5
BIC (smaller is better)	433.2

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
age	3	75	37.60	<.0001
sex	1	25	9.28	0.0054
age*sex	3	75	2.51	0.0649

Continued

MODEL III

(The output was edited to remove Class Level Information and Number of Observations (both same as models I and II) and Iteration History (convergence criteria were met).)

The Mixed Procedure

Model Information

Dependent Variable	distance
Covariance Structure	Unstructured
Subject Effect	subject(sex)
Estimation Method	REML
Residual Variance Method	None
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

Dimensions

Covariance Parameters	10
Columns in X	15
Columns in Z	0
Subjects	27
Max Obs Per Subject	4

Estimated R Correlation Matrix for subject(sex) F01 Female

Row	Col1	Col2	Col3	Col4
1	1.0000	0.5707	0.6613	0.5216
2	0.5707	1.0000	0.5632	0.7262
3	0.6613	0.5632	1.0000	0.7281
4	0.5216	0.7262	0.7281	1.0000

Fit Statistics

-2 Res Log Likelihood	414.0
AIC (smaller is better)	xxxxx
AICC (smaller is better)	436.5
BIC (smaller is better)	447.0

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
9	56.46	<.0001

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
age	3	25	34.45	<.0001
sex	1	25	9.29	0.0054
age*sex	3	25	2.93	0.0532

Continued

(a) (1 mark) The models include the interaction of sex and age. Explain in practical terms why this was included in the models.

(b) (2 marks) The model was fit using the mixed models procedure in SAS. Explain why the model is "mixed".

(c) (4 marks) Write the model that was fit in model I, carefully defining all terms. (Do not write the fitted equation; write the model in terms of its parameters.)

(d) (2 marks) For model I, why is the number of covariance parameters equal to 2?

(e) (1 mark) What is the value of AIC for model III?

(f) (2 marks) AR(1) is a commonly used covariance structure in situations such as this, where observations are taken over time. Explain why it is not an appropriate covariance structure for these data by comparing at least 2 different kinds of information given in the SAS output.

(g) (2 marks) How do the conditions for valid inference for this model differ from the conditions needed for a multiple linear regression model?

5. (a) (6 marks) In order for inferences to be valid, conditions must be met. Assume standard analyses that were taught in this course are being carried out.

   i. Give two examples of conditions that must be met for both analysis of variance and binomial logistic regression models in order for the inferences to be valid.

   ii. Give two examples of conditions that must be met for the inferences to be valid for an analysis of variance model but which are not necessary for a binomial logistic regression model.

   iii. Give two examples of conditions that must be met for the inferences to be valid for a binomial logistic regression model but which are not necessary for an analysis of variance model.

   (b) (4 marks) Here are two recent quotes from lecture.

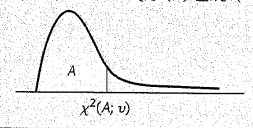
   "What does it mean if you make predictions from a fitted model that does not adequately describe the data?"

   "Only do inference on valid models."

   Imagine it is sometime in the future and you have been hired to do the statistical analysis on the data collected from a scientific study. How will the ideas behind these quotes affect the work you will do? And why is this important?

TABLE B.3 **Percentiles of the χ^2 Distribution.**

Entry is $\chi^2(A;\nu)$ where $P\{\chi^2(\nu) \leq \chi^2(A;\nu)\} = A$



	A									
ν	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.0^4393	0.0^3157	0.0^3982	0.0^2393	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Continued

Some formulae:

Pooled t-test

$$t_{obs} = \frac{\overline{y}_1 - \overline{y}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

Test for two proportions

$$z_{obs} = (\hat{\pi}_1 - \hat{\pi}_2) \Big/ \sqrt{\hat{\pi}_p(1-\hat{\pi}_p)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

Linear Regression

$$b_1 = \frac{\sum(X_i-\overline{X})(Y_i-\overline{Y})}{\sum(X_i-\overline{X})^2} = \frac{\sum X_iY_i - n\overline{XY}}{\sum X_i^2 - n\overline{X}^2} \qquad b_0 = \overline{Y} - b_1\overline{X}$$

One-way analysis of variance

$$\text{SSTO} = \sum_{i=1}^{N}(Y_i - \overline{Y})^2 \qquad \text{SSE} = \sum_{g=1}^{G}\sum_{(g)}(Y_i - \overline{Y}_g)^2 \qquad \text{SSR} = \sum_{g=1}^{G} n_g(\overline{Y}_g - \overline{Y})^2$$

Bernoulli and Binomial distributions

If $Y \sim$ Bernoulli(π)
$\mathrm{E}(Y) = \pi$, $\mathrm{Var}(Y) = \pi(1-\pi)$

If $Y \sim$ Binomial(m, π)
$\mathrm{E}(Y) = m\pi$, $\mathrm{Var}(Y) = m\pi(1-\pi)$

Logistic Regression with Binomial Response formulae

$$\text{Deviance} = 2\sum_{i=1}^{n}\{y_i\log(y_i) + (m_i - y_i)\log(m_i - y_1) - y_i\log(\hat{y}_i) + (m_i - y_i)\log(m_i - \hat{y}_1)\}$$

$$D_{res,i} = \text{sign}(y_i - m_i\hat{\pi}_i)\sqrt{2\left\{y_i\log\left(\frac{y_i}{m_i\hat{\pi}_i}\right) + (m_i - y_i)\log\left(\frac{m_i - y_i}{m_i - m_i\hat{\pi}_i}\right)\right\}}$$

$$P_{res,i} = \frac{y_i - m_i\hat{\pi}_i}{\sqrt{m_i\hat{\pi}_i(1-\hat{\pi}_i)}}$$

Multinomial distribution for 2×2 table

$$\Pr(\mathbf{Y} = \mathbf{y}) = \frac{n!}{y_{11}!y_{12}!y_{21}!y_{22}!}\pi_{11}^{y_{11}}\pi_{12}^{y_{12}}\pi_{21}^{y_{21}}\pi_{22}^{y_{22}}$$

Poisson distribution

$$\Pr(Y = y) = \frac{\mu^y e^{-\mu}}{y!},\ y = 0, 1, 2, \ldots$$
$$\mathrm{E}(Y) = \mu,\ \mathrm{Var}(Y) = \mu$$

Two-way contingency tables (easily generalizable to three-way tables)

$$X^2 = \sum_{j=1}^{J}\sum_{i=1}^{I}\frac{(y_{ij} - \hat{\mu}_{ij})^2}{\hat{\mu}_{ij}} \qquad G^2 = 2\sum_{j=1}^{J}\sum_{i=1}^{I} y_{ij}\log\left(\frac{y_{ij}}{\hat{\mu}_{ij}}\right)$$

$$D_{res,ij} = \text{sign}(y_{ij} - \hat{\mu}_{ij})\sqrt{2\left\{y_{ij}\log\left(\frac{y_{ij}}{\hat{\mu}_{ij}}\right) - y_{ij} + \hat{\mu}_{ij}\right\}}$$

$$P_{res,ij} = \frac{y_{ij} - \hat{\mu}_{ij}}{\sqrt{\hat{\mu}_{ij}}}$$

Model Fitting Criteria

$$\text{AIC} = -2\log(L) + 2p \qquad \text{SC} = -2\log(L) + p\log(N)$$