STA 302 H1F / 1001 HF – Fall 2010
Test

October 21, 2010

LAST NAME:________________________FIRST NAME:____________________________

STUDENT NUMBER:____

INSTRUCTIONS:
- Time: 90 minutes
- Aids allowed: calculator.
- All of the formulae below can be taken as known unless a question indicates otherwise.
- Total points: 50

Some formulae:

$$b_1 = \frac{\sum(x_i-\overline{x})(y_i-\overline{y})}{\sum(x_i-\overline{x})^2} = \frac{\sum x_iy_i - n\overline{x}\overline{y}}{\sum x_i^2 - n\overline{x}^2} \qquad b_0 = \overline{y} - b_1\overline{x}$$

$$\mathrm{Var}(\hat{\beta}_1 \,|\, X) = \frac{\sigma^2}{\sum(x_i-\overline{x})^2} \qquad \mathrm{Var}(\hat{\beta}_0 \,|\, X) = \sigma^2\left(\frac{1}{n} + \frac{\overline{x}^2}{\sum(x_i-\overline{x})^2}\right)$$

$$\mathrm{Cov}(\hat{\beta}_0, \hat{\beta}_1 \,|\, X) = -\frac{\sigma^2\overline{x}}{\sum(x_i-\overline{x})^2} \qquad \mathrm{SST} = \sum(y_i - \overline{y})^2$$

$$\mathrm{RSS} = \sum(y_i - \hat{y}_i)^2 \qquad \mathrm{SSReg} = b_1^2\sum(x_i - \overline{x})^2 = \sum(\hat{y}_i - \overline{y})^2$$

$$\mathrm{Var}(\hat{y}|X = x^*) = \sigma^2\left(\frac{1}{n} + \frac{(x^*-\overline{x})^2}{\sum(x_i-\overline{x})^2}\right) \quad \mathrm{Var}(Y - \hat{y}|X = x^*) = \sigma^2\left(1 + \frac{1}{n} + \frac{(x^*-\overline{x})^2}{\sum(x_i-\overline{x})^2}\right)$$

$$r = \frac{\sum(x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\sum(x_i-\overline{x})^2\sum(y_i-\overline{y})^2}} \qquad SXX = \sum(x_i - \overline{x})^2 = \sum x_i^2 - n\overline{x}^2$$

$$h_{ij} = \frac{1}{n} + \frac{(x_i-\overline{x})(x_j-\overline{x})}{SXX} \quad \left(h_{ii} > \frac{4}{n}\right) \qquad \mathrm{DFBETAS}_{ik} = \frac{b_k - b_{k(i)}}{s.e.(b_k)} \quad \left(> 1 \text{ or } \frac{2}{\sqrt{n}}\right)$$

$$\mathrm{DFFITS}_i = \frac{\hat{y}_i - \hat{y}_{i(i)}}{s.e.(\hat{y}_i)} \quad \left(> 1 \text{ or } 2\sqrt{\frac{2}{n}}\right) \qquad D_i = \frac{\sum(\hat{y}_{j(i)} - \hat{y}_j)^2}{2S^2} \quad \left(> \frac{4}{n-2}\right)$$

1abc	1d	2	3abc	3def	3gh, 4

1. Consider the example we have been examining in lecture in which we have been using simple linear regression to model how the atmospheric concentrations of CFC-11 (in parts per trillion) were changing as a function of time (in years) before the implementation of the Montreal Protocol. The data consist of 153 measurements of CFC-11 taken monthly from 1977 to the end of 1989 and the date on which the measurements were taken.

   (a) (2 marks) State the simple linear regression model being used. Which terms in the model are random variables?

   (b) (3 marks) State the Gauss-Markov conditions for the model in part (a).

   (c) (4 marks) Assume that the Gauss-Markov conditions and the usual distributional assumptions hold. State fully the distributions of the random variables in the model. How do the distributions change with time?

(Question 1 continued.)

(d) Suppose that the regression model has been fit to the data and the usual statistics have been calculated.

i. (3 marks) Show that $\sum_{i=1}^{n} \hat{e}_i x_i = 0$.

ii. (3 marks) Show that estimator of the slope of the regression line is unbiased.

2. Suppose a simple linear regression is carried out to investigate the relationship between a dependent variable Y and an independent variable X. The data consist of n pairs of observed values of X and Y, (x_i, y_i), $i = 1, \ldots, n$.

   (a) (2 marks) What is the first step you should carry out in the regression analysis? What do you hope to accomplish in this step?

   (b) (1 mark) As part of the output for the simple linear regression analysis, SAS gives the results of the statistical test with null hypothesis $H_0 : \beta_1 = 0$ and alternative hypothesis $H_a : \beta_1 \neq 0$. Why is this test of particular interest?

   (c) You suspect that there is a strong linear relationship between Y and X.

      i. (2 marks) For the test with null hypothesis $H_0 : \beta_1 = 0$, do you expect the test statistic to be large or small? Explain.

      ii. (2 marks) For the test with null hypothesis $H_0 : \beta_1 = 0$, do you expect the p-value to be large or small? Explain.

3. In a paper published in the *British Medical Journal* in 1965, Lea looked at data from counties in regions of Great Britain, Norway, and Sweden. He was interested in how the mean annual temperature (in degrees Fahrenheit) affected the mortality index for breast cancer. (The mortality index is a measure of the death rate for women diagnosed with breast cancer. The index Lea used measures death rate relative to the average death rate for England and Wales. On his scale, England and Wales was given the value of 100. Mortality indices greater than 100 indicate a higher death rate than that of England and Wales.)

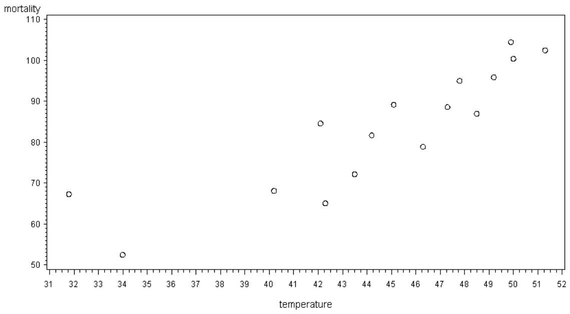
   Here are some quantiles from t-distributions which may be useful for some of the questions that follow.

Degrees of	Upper-tail probability				
freedom	0.005	0.010	0.025	0.05	0.10
14	2.977	2.624	2.145	1.761	1.345
15	2.947	2.602	2.131	1.753	1.341
16	2.921	2.583	2.120	1.746	1.337

   Some SAS output is given below and on the next page for the analysis Lea carried out.

```
                                The REG Procedure

                           Descriptive Statistics
                                       Uncorrected                     Standard
Variable              Sum         Mean          SS      Variance      Deviation
Intercept        16.00000      1.00000    16.00000             0              0
temperature     713.50000     44.59375       32285      31.17663        5.58360
mortality      1333.50000     83.34375      114535     226.42929       15.04757

                        Dependent Variable: mortality

                            Analysis of Variance
                                    Sum of          Mean
Source                   DF        Squares        Square    F Value    Pr > F
Model                     1     2599.53358    2599.53358        (A)    <.0001
Error                    14      796.90580      56.92184
Corrected Total          15     3396.43938

             Root MSE              7.54466    R-Square     0.7654
             Dependent Mean       83.34375    Adj R-Sq     0.7486
             Coeff Var             9.05246

                            Parameter Estimates
                            Parameter       Standard
    Variable       DF        Estimate          Error    t Value    Pr > |t|
    Intercept       1       -21.79469       15.67190      -1.39      0.1860
    temperature     1         2.35769        0.34888        (B)      <.0001
```

```
                             Output Statistics
      Dependent   Predicted     Std Error               Std Error   Student
Obs    Variable       Value  Mean Predict     Residual   Residual  Residual
  1    102.5000     99.1550        3.0053       3.3450      6.920     0.483
  2    104.5000     95.8543        2.6429       8.6457      7.067     1.223
  3    100.4000     96.0900        2.6674       4.3100      7.057     0.611
  4     95.9000     94.2039        2.4779       1.6961      7.126     0.238
  5     87.0000     92.5535        2.3270      -5.5535      7.177    -0.774
  6     95.0000     90.9031        2.1929       4.0969      7.219     0.568
  7     88.6000     89.7243        2.1093      -1.1243      7.244    -0.155
  8     89.2000     84.5373        1.8944       4.6627      7.303     0.638
  9     78.9000     87.3666        1.9779      -8.4666      7.281    -1.163
 10     84.6000     77.4642        2.0772       7.1358      7.253     0.984
 11     81.7000     82.4154        1.8912      -0.7154      7.304   -0.0980
 12     72.2000     80.7650        1.9244      -8.5650      7.295    -1.174
 13     65.1000     77.9358        2.0489     -12.8358      7.261    -1.768
 14     68.1000     72.9846        2.4305      -4.8846      7.142    -0.684
 15     67.3000     53.1800        4.8457      14.1200      5.783     2.442
 16     52.5000     58.3669        4.1494      -5.8669      6.301    -0.931

                            Cook's              Hat Diag       Cov
Obs     -2-1 0 1 2               D    RStudent         H     Ratio    DFFITS
  1   |      |      |        0.022      0.4697    0.1587    1.3329    0.2040
  2   |      |**    |        0.105      1.2475    0.1227    1.0544    0.4666
  3   |      |*     |        0.027      0.5965    0.1250    1.2558    0.2254
  4   |      |      |        0.003      0.2298    0.1079    1.2895    0.0799
  5   |     *|      |        0.031     -0.7621    0.0951    1.1744   -0.2471
  6   |      |*     |        0.015      0.5533    0.0845    1.2092    0.1681
  7   |      |      |        0.001     -0.1497    0.0782    1.2538   -0.0436
  8   |      |*     |        0.014      0.6244    0.0630    1.1668    0.1620
  9   |    **|      |        0.050     -1.1789    0.0687    1.0164   -0.3203
 10   |      |*     |        0.040      0.9826    0.0758    1.0874    0.2814
 11   |      |      |        0.000     -0.0944    0.0628    1.2358   -0.0244
 12   |    **|      |        0.048     -1.1915    0.0651    1.0082   -0.3143
 13   |   ***|      |        0.124     -1.9327    0.0738    0.7555   -0.5454
 14   |     *|      |        0.027     -0.6703    0.1038    1.2090   -0.2281
 15   |      |****  |        2.093      3.1052    0.4125    0.6507    2.6020
 16   |     *|      |        0.188     -0.9264    0.3025    1.4632   -0.6100
```



Questions related to this output begin on the next page.

(Question 3 continued.)

(a) (5 marks) What are the values of each of the following:

- the number of observations ________________

- the number replaced by (A) in the SAS output ________________

- the number replaced by (B) in the SAS output ________________

- the estimate of the correlation between
mortality index and mean annual temperature ________________

- the estimate of the variance of the error ________________

(b) (4 marks) Give two different numbers from the SAS output that give some indication of the strength of the linear relationship between mortality index and mean annual temperature. For each number, state what it measures. Do not choose numbers that are missing from the output and do not choose two numbers that are equal.

(c) For the test with p-value 0.1860:

i. (1 mark) What are the null and alternative hypotheses?

ii. (2 marks) What do you conclude? State your conclusion in the practical context of the data being analysed.

(Question 3 continued.)

(d) (3 marks) Calculate a 90% confidence interval for the slope. How is it related to your answers to parts (b) and/or (c)?

(e) (2 marks) DFFITS is given in the output statistics and its formula is given on the first page. Explain what it measures.

(f) (4 marks) Based on the given output statistics, what concerns do you have about the fit of the regression line to the data? Give at least two numbers in the output that indicate that this concern exists. Draw a sketch that illustrates the implications of your concern.

(g) (4 marks) Calculate a 95% prediction interval for the predicted mortality rate for a county with a mean annual temperature of 37 degrees Fahrenheit.

(h) (1 mark) How does your answer to part (f) affect your interpretation of the prediction interval in part (g)?

4. (2 marks) A study was carried out to examine the effect of taking Vitamin D tablets on levels of LDL in the blood. (LDL is "bad" cholesterol and is measured in mg/dL.) Thirty subjects who were taking no medication for their cholesterol were recruited into the study and their LDL was measured. They then took Vitamin D tablets for 30 days while otherwise maintaining their usual diet and their LDL was measured again. A two-sided t-test with null hypothesis that the mean is 0 for the before-after change in LDL had p-value 0.9207. The researchers also fit a simple linear regression model with response variable LDL after 30 days and explanatory variable LDL at the start of the study. The fitted regression line had intercept 13.0 and slope 0.887. The p-value was < 0.001 for the two-sided test with null hypothesis that the slope is 0. The researchers concluded the following: "On average, LDL did not differ with Vitamin D intake, but, importantly, for subjects with higher LDL values than the population average of 115 mg/dL, LDL levels were lower on average after taking Vitamin D with greater reductions for patients with higher initial LDL levels."

   Is the researchers' conclusion supported by the analysis? Why or why not?