STA 302 H1F / 1001 HF – Fall 2009
Test
October 22, 2009

LAST NAME:________________________FIRST NAME:______________________________

STUDENT NUMBER:____

ENROLLED IN: (circle one)   STA 302   STA 1001

INSTRUCTIONS:
- Time: 90 minutes
- Aids allowed: calculator.
- A table of values from the t distribution is on the last page (page 10).
- Total points: 50

Some formulae:

$$b_1 = \frac{\sum(X_i-\overline{X})(Y_i-\overline{Y})}{\sum(X_i-\overline{X})^2} = \frac{\sum X_iY_i - n\overline{X}\overline{Y}}{\sum X_i^2 - n\overline{X}^2} \qquad b_0 = \overline{Y} - b_1\overline{X}$$

$$\mathrm{Var}(b_1) = \frac{\sigma^2}{\sum(X_i-\overline{X})^2} \qquad \mathrm{Var}(b_0) = \sigma^2\left(\frac{1}{n} + \frac{\overline{X}^2}{\sum(X_i-\overline{X})^2}\right)$$

$$\mathrm{Cov}(b_0, b_1) = -\frac{\sigma^2\overline{X}}{\sum(X_i-\overline{X})^2} \qquad \mathrm{SSTO} = \sum(Y_i - \overline{Y})^2$$

$$\mathrm{SSE} = \sum(Y_i - \hat{Y}_i)^2 \qquad \mathrm{SSR} = b_1^2\sum(X_i - \overline{X})^2 = \sum(\hat{Y}_i - \overline{Y})^2$$

$$\sigma^2\{\hat{Y}_h\} = \mathrm{Var}(\hat{Y}_h) = \sigma^2\left(\frac{1}{n} + \frac{(X_h-\overline{X})^2}{\sum(X_i-\overline{X})^2}\right) \qquad \sigma^2\{\mathrm{pred}\} = \mathrm{Var}(Y_h - \hat{Y}_h) = \sigma^2\left(1 + \frac{1}{n} + \frac{(X_h-\overline{X})^2}{\sum(X_i-\overline{X})^2}\right)$$

$$r = \frac{\sum(X_i-\overline{X})(Y_i-\overline{Y})}{\sqrt{\sum(X_i-\overline{X})^2\sum(Y_i-\overline{Y})^2}} \qquad S_{XX} = \sum(X_i - \overline{X})^2 = \sum X_i^2 - n\overline{X}^2$$

1abc	1de	2a	2bcd	2efgh	3ab	3c	3de

1. The method of least squares is used to fit a simple linear regression model $Y = \beta_0 + \beta_1 X + \epsilon$ to n observations (X_i, Y_i). The values of the X_i's are not realizations of random variables but are fixed in advance by the researcher. Assume the following: the form of the model is appropriate, the Gauss-Markov conditions hold, and the distribution of the error terms is Normal. In your answers, you may use any of the formulae on the front page.

   (a) (4 marks) Which of the assumptions are needed to fit the model using least squares? How would you assess the necessary assumptions?

   (b) (3 marks) What is $E(Y)$? Which of the assumptions did you use to determine your answer?

   (c) (2 marks) Suppose the researcher is interested in the relationship between X and Y on a certain range of X's. She uses the smallest value in the range of X for half of the observations and the largest value in the range of X for the other half of the observations and fits the simple linear regression model to the resulting data. What is the advantage of fixing the X's to be these values? What is the disadvantage?

(Question 1 continued)

(d) (4 marks) Show that the least squares estimator b_1 is an unbiased estimator of β_1.

(e) (2 marks) Are b_0 (the estimator of the intercept) and b_1 (the estimator of the slope) independent? Explain.

2. CFC-11 atmospheric concentrations in parts per trillion were measured monthly. The following SAS output shows the results of the regression of atmospheric concentration on time (in years) for the period 1977 to 1989. On the next page is SAS output for the regression for the period 1995 to 2004. Some of the output has been removed and, in the first regression, some of the numerical values have been replaced by letters. Answer the questions assuming that the usual regression model assumptions hold.

```
              Before Montreal Protocol (before January 1990)

                               Descriptive Statistics

                                                 Uncorrected                      Standard
Variable             Sum            Mean                  SS       Variance      Deviation
Intercept      153.00000         1.00000           153.00000              0              0
time              303466      1983.43464           601906139       14.16876        3.76414
cfc11              30286       197.94771             6199037     1342.05751       36.63410


                               The REG Procedure
                             Dependent Variable: cfc11

                                   Analysis of Variance
                                    Sum of           Mean
Source                   DF        Squares         Square    F Value    Pr > F
Model                   (A)         203119         203119        (B)    <.0001
Error                   151      874.00094        5.78809
Corrected Total         (C)         203993

             Root MSE                  (D)     R-Square   0.9957
             Dependent Mean      197.94771

                              Parameter Estimates
                               Parameter       Standard
     Variable       DF          Estimate          Error     t Value     Pr > |t|
     Intercept       1            -19064            (E)     -185.40       <.0001
     time            1           9.71152        0.05184      187.33       <.0001
```

(a) (5 marks) Find the values of the numbers that have been replaced by letters:

(A) = ________________________

(B) = ________________________

(C) = ________________________

(D) = ________________________

(E) = ________________________

(Question 2 continued)

After Montreal Protocol (after December 1994)

Descriptive Statistics

Variable	Sum	Mean	Uncorrected SS	Variance	Standard Deviation
Intercept	116.00000	1.00000	116.00000	0	0
time	231985	1999.86710	463939247	7.92451	2.81505
cfc11	30641	264.14741	8096893	27.17849	5.21330

The REG Procedure
Dependent Variable: cfc11

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	3061.55314	3061.55314	5455.70	<.0001
Error	114	63.97289	0.56117		
Corrected Total	115	3125.52602			

Root MSE	0.74911
Dependent Mean	264.14741

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	3929.67750	49.62630	79.19	<.0001
time	1	-1.83289	0.02481	-73.86	<.0001

(b) (2 marks) Calculate a 90% confidence interval for the intercept for the regression for the period 1995 to 2004 (after the Montreal Protocol).

(c) (2 marks) For the regression for the time period 1995 to 2004, find R^2 and explain what it measures.

(d) (1 mark) Interpret the estimated slope in practical terms.

(e) (4 marks) Carry out an hypothesis test to determine whether the slopes of the lines for the regressions for the two time periods differ. If you do not have all the information you need to completely answer the question, indicate what is missing and give the most complete answer you can.

(f) (4 marks) Use one of the fitted models to predict what the atmospheric concentration of CFC-11 on October 1, 2009 was (when `time` = 2009.75) and give a 99% interval for your prediction.

(g) (2 marks) Do you feel confident that the actual concentration of CFC-11 measured on October 1, 2009 is in the interval you calculated in part (f)? Why or why not?

(h) (2 marks) Using only what you know about how the data were collected, does it seem possible that there are any violations in the Gauss-Markov conditions for these regressions? Explain.

3. Golf tournaments take place over a few days. On each day of the tournament one round of golf is played. In this question, we are looking at the relationship between golfers' scores on the first round and their scores on the second round in the 2000 British Open. In golf, low scores are good. Some output from SAS is given below.

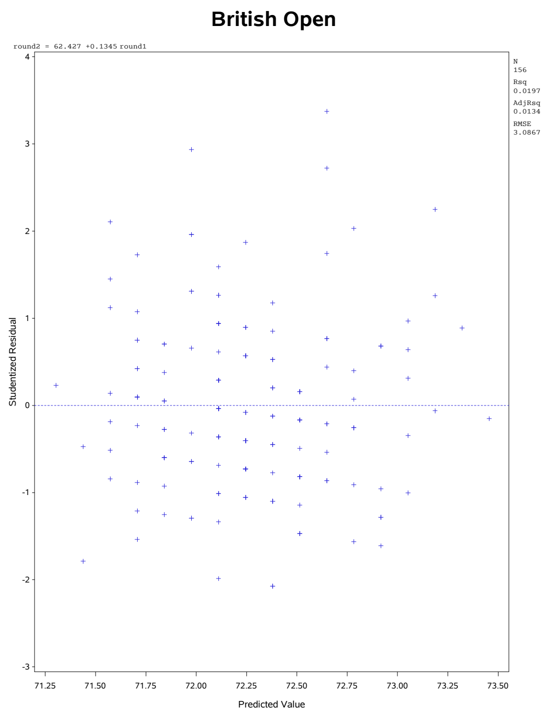
```
                         The REG Procedure
                    Dependent Variable: round2

                        Analysis of Variance
                                  Sum of           Mean
Source                   DF      Squares         Square    F Value    Pr > F
Model                     1     29.51923       29.51923       3.10    0.0804
Error                   154   1467.22436        9.52743
Corrected Total         155   1496.74359

             Root MSE              3.08665    R-Square     0.0197
             Dependent Mean       72.24359    Adj R-Sq     0.0134
             Coeff Var             4.27256

                        Parameter Estimates
                           Parameter       Standard
     Variable      DF       Estimate          Error    t Value    Pr > |t|
     Intercept      1       62.42741        5.58218      11.18      <.0001
     round1         1        0.13449        0.07641       1.76      0.0804
```

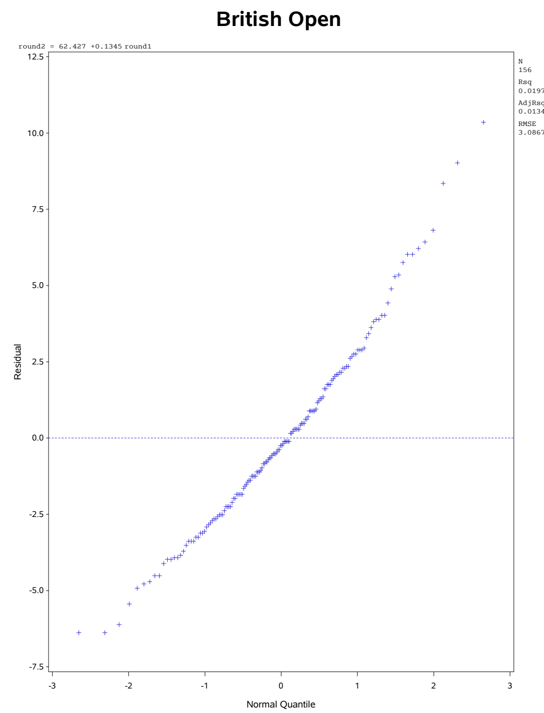
(a) (2 marks) Is there evidence of a linear relationship between golf scores on the first and second round of the tournament? Explain.

(b) (3 marks) The lowest score obtained on the first round was 66. Predict the second round score of the golfer who achieved this. Is this surprising? Explain your answer in terms of known facts about simple linear regression.

(Question 3 continued)

(c) (4 marks) Below are plots of the studentized residuals versus the predicted values, and a normal quantile plot of the residuals. What additional information do you learn from the plots? Be specific.





(Question 3 continued)

(d) (2 marks) In assignment 1 we considered the relationship between football kickers' field goal percentages one year with the percentage of field goals scored the previous year and found problems with violations of the Gauss-Markov conditions in the initial analysis. In the regression here we are examining the relationship between golf scores on one round with golf scores on the previous round. Do we have a similar problem with violations of the Gauss-Markov assumptions? Why or why not?

(e) (2 marks) The plot of the data below includes 95% confidence intervals for the mean score in round 2 given the score in round 1. About 90% of the data points fall outside the confidence limits. Explain how it can occur that so many observations are missed.

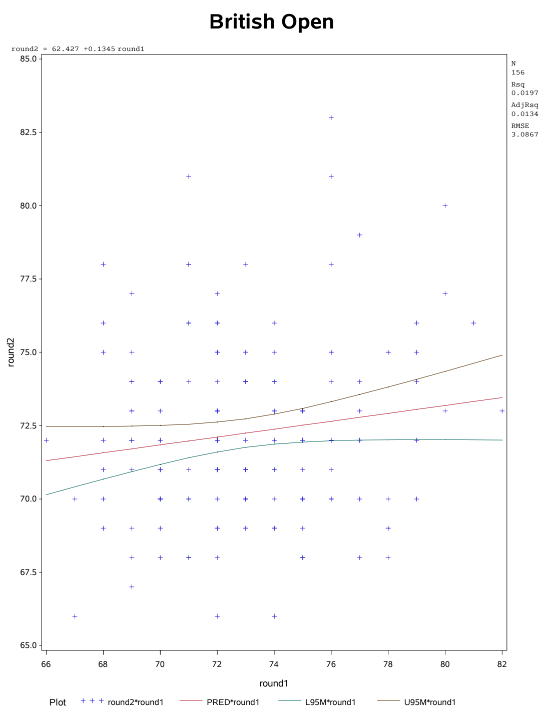
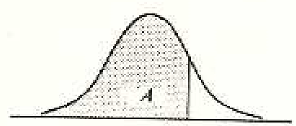


TABLE B.2 Percentiles of the t Distribution.

Entry is $t(A;\nu)$ where $P\{t(\nu) \le t(A;\nu)\} = A$



ν	A: .60	.70	.80	.85	.90	.95	.975
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

TABLE B.2 *(concluded)* Percentiles of the t Distribution.

ν	A: .98	.985	.99	.9925	.995	.9975	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291