STA 302 H1F / 1001 HF – Fall 2009 Test October 22, 2009

| LAST NAME: | FIRST | Г NAME: | |
|---------------------------|---------|----------|--|
| STUDENT NUMBER: | | | |
| ENROLLED IN: (circle one) | STA 302 | STA 1001 | |
| INSTRUCTIONS: | | | |

- Time: 90 minutes
- Aids allowed: calculator.
- A table of values from the t distribution is on the last page (page 10).
- Total points: 50

Some formulae:

$$b_{1} = \frac{\sum(X_{i} - \overline{X})(Y_{i} - \overline{Y})}{\sum(X_{i} - \overline{X})^{2}} = \frac{\sum X_{i}Y_{i} - n\overline{X}\overline{Y}}{\sum X_{i}^{2} - n\overline{X}^{2}} \qquad b_{0} = \overline{Y} - b_{1}\overline{X}$$

$$\operatorname{Var}(b_{1}) = \frac{\sigma^{2}}{\sum(X_{i} - \overline{X})^{2}} \qquad \operatorname{Var}(b_{0}) = \sigma^{2}\left(\frac{1}{n} + \frac{\overline{X}^{2}}{\sum(X_{i} - \overline{X})^{2}}\right)$$

$$\operatorname{Cov}(b_{0}, b_{1}) = -\frac{\sigma^{2}\overline{X}}{\sum(X_{i} - \overline{X})^{2}} \qquad \operatorname{SSTO} = \sum(Y_{i} - \overline{Y})^{2}$$

$$\operatorname{SSE} = \sum(Y_{i} - \hat{Y}_{i})^{2} \qquad \operatorname{SSR} = b_{1}^{2}\sum(X_{i} - \overline{X})^{2} = \sum(\hat{Y}_{i} - \overline{Y})^{2}$$

$$\sigma^{2}\{\hat{Y}_{h}\} = \operatorname{Var}(\hat{Y}_{h}) = \sigma^{2}\left(\frac{1}{n} + \frac{(X_{h} - \overline{X})^{2}}{\sum(X_{i} - \overline{X})^{2}}\right) \qquad \sigma^{2}\{\operatorname{pred}\} = \operatorname{Var}(Y_{h} - \hat{Y}_{h}) = \sigma^{2}\left(1 + \frac{1}{n} + \frac{(X_{h} - \overline{X})^{2}}{\sum(X_{i} - \overline{X})^{2}}\right)$$

$$r = \frac{\sum(X_{i} - \overline{X})(Y_{i} - \overline{Y})^{2}}{\sqrt{\sum(X_{i} - \overline{X})^{2}}\sum(Y_{i} - \overline{Y})^{2}} \qquad S_{XX} = \sum(X_{i} - \overline{X})^{2} = \sum X_{i}^{2} - n\overline{X}^{2}$$

| 1abc | 1de | 2a | 2bcd | 2efgh | 3ab | 3c | 3de |
|------|-----|----|------|-------|-----|----|-----|
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

- 1. The method of least squares is used to fit a simple linear regression model $Y = \beta_0 + \beta_1 X + \epsilon$ to *n* observations (X_i, Y_i) . The values of the X_i 's are not realizations of random variables but are fixed in advance by the researcher. Assume the following: the form of the model is appropriate, the Gauss-Markov conditions hold, and the distribution of the error terms is Normal. In your answers, you may use any of the formulae on the front page.
 - (a) (4 marks) Which of the assumptions are needed to fit the model using least squares? How would you assess the necessary assumptions?

(b) (3 marks) What is E(Y)? Which of the assumptions did you use to determine your answer?

(c) (2 marks) Suppose the researcher is interested in the relationship between X and Y on a certain range of X's. She uses the smallest value in the range of X for half of the observations and the largest value in the range of X for the other half of the observations and fits the simple linear regression model to the resulting data. What is the advantage of fixing the X's to be these values? What is the disadvantage?

(Question 1 continued)

(d) (4 marks) Show that the least squares estimator b_1 is an unbiased estimator of β_1 .

(e) (2 marks) Are b_0 (the estimator of the intercept) and b_1 (the estimator of the slope) independent? Explain.

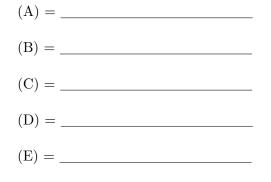
2. CFC-11 atmospheric concentrations in parts per trillion were measured monthly. The following SAS output shows the results of the regression of atmospheric concentration on time (in years) for the period 1977 to 1989. On the next page is SAS output for the regression for the period 1995 to 2004. Some of the output has been removed and, in the first regression, some of the numerical values have been replaced by letters. Answer the questions assuming that the usual regression model assumptions hold.

Before Montreal Protocol (before January 1990)

Descriptive Statistics

| | | | | Uncorre | ected | | | Standard |
|--------------|-----------|------|------------|-----------|------------|---------|-------|-----------|
| Variable | Su | m | Mean | | SS | Varia | nce | Deviation |
| Intercept | 153.0000 | 0 | 1.00000 | 153.0 | 00000 | | 0 | 0 |
| time | 30346 | 6 1 | 1983.43464 | 60190 | 06139 | 14.16 | 876 | 3.76414 |
| cfc11 | 3028 | 6 | 197.94771 | 619 | 99037 | 1342.05 | 751 | 36.63410 |
| | | | | | | | | |
| | | | | | | | | |
| | | | The REG | Procedu | re | | | |
| | | | Dependent | Variable | cfc11 | | | |
| | | | | | | | | |
| | | | An | alysis of | f Varian | ce | | |
| | | | Su | m of | M | ean | | |
| Source | | DF | Squ | ares | Squ | are F | Value | Pr > F |
| Model | | (A) | 20 | 3119 | 203 | 119 | (B) | <.0001 |
| Error | | 151 | 874.0 | 0094 | 5.78 | 809 | | |
| Corrected To | otal | (C) | 20 | 3993 | | | | |
| | | | , | | | 0 0057 | | |
| | Root MSE | | - | | R-Square | 0.9957 | | |
| | Dependent | Mean | 197.9 | 4771 | | | | |
| | | | Parameter | Estimate | es | | | |
| | | Pa | arameter | Stand | lard | | | |
| Variab | Le DF | | Estimate | | | t Value | Pr > | ltl |
| Interce | | - | -19064 | | | -185.40 | | 0001 |
| time | 1 | | 9.71152 | | -, 5184 | 187.33 | | 0001 |

(a) (5 marks) Find the values of the numbers that have been replaced by letters:



(Question 2 continued)

After Montreal Protocol (after December 1994)

Descriptive Statistics

| | | | Uncorrected | | Standard |
|-----------|-----------|------------|-------------|----------|-----------|
| Variable | Sum | Mean | SS | Variance | Deviation |
| Intercept | 116.00000 | 1.00000 | 116.00000 | 0 | 0 |
| time | 231985 | 1999.86710 | 463939247 | 7.92451 | 2.81505 |
| cfc11 | 30641 | 264.14741 | 8096893 | 27.17849 | 5.21330 |

The REG Procedure Dependent Variable: cfc11

| | | Analysis of | Variance | | |
|-----------------|-----|-------------|------------|---------|--------|
| | | Sum of | Mean | | |
| Source | DF | Squares | Square | F Value | Pr > F |
| Model | 1 | 3061.55314 | 3061.55314 | 5455.70 | <.0001 |
| Error | 114 | 63.97289 | 0.56117 | | |
| Corrected Total | 115 | 3125.52602 | | | |
| | | | | | |

| Root MSE | 0.74911 |
|----------------|-----------|
| Dependent Mean | 264.14741 |

| | | Parameter | Estimates | | |
|-----------|----|------------|-----------|---------|---------|
| | | Parameter | Standard | | |
| Variable | DF | Estimate | Error | t Value | Pr > t |
| Intercept | 1 | 3929.67750 | 49.62630 | 79.19 | <.0001 |
| time | 1 | -1.83289 | 0.02481 | -73.86 | <.0001 |

- (b) (2 marks) Calculate a 90% confidence interval for the intercept for the regression for the period 1995 to 2004 (after the Montreal Protocol).
- (c) (2 marks) For the regression for the time period 1995 to 2004, find \mathbb{R}^2 and explain what it measures.

(d) (1 mark) Interpret the estimated slope in practical terms.

(Question 2 continued)

(e) (4 marks) Carry out an hypothesis test to determine whether the slopes of the lines for the regressions for the two time periods differ. If you do not have all the information you need to completely answer the question, indicate what is missing and give the most complete answer you can.

(f) (4 marks) Use one of the fitted models to predict what the atmospheric concentration of CFC-11 on October 1, 2009 was (when time = 2009.75) and give a 99% interval for your prediction.

- (g) (2 marks) Do you feel confident that the actual concentration of CFC-11 measured on October 1, 2009 is in the interval you calculated in part (f)? Why or why not?
- (h) (2 marks) Using only what you know about how the data were collected, does it seem possible that there are any violations in the Gauss-Markov conditions for these regressions? Explain.

3. Golf tournaments take place over a few days. On each day of the tournament one round of golf is played. In this question, we are looking at the relationship between golfers' scores on the first round and their scores on the second round in the 2000 British Open. In golf, low scores are good. Some output from SAS is given below.

The REG Procedure Dependent Variable: round2 Analysis of Variance Sum of Mean DF Squares Square F Value 1 29.51923 29.51923

Pr > F

0.0804

3.10

Source

Model

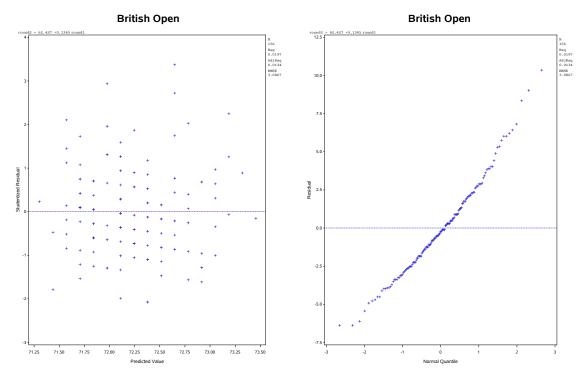
| Error | | 154 | 1467.224 | 136 | 9.5274 | 43 | |
|----------------|-----------|------|-------------|-----------|--------|-------|---------|
| Corrected Tota | al | 155 | 1496.743 | 359 | | | |
| | | | | | | | |
| F | Root MSE | | 3.086 | 665 R-S | Square | 0.01 | .97 |
| Ľ | Dependent | Mean | 72.243 | 359 Ad | j R-Sq | 0.01 | .34 |
| C | Coeff Var | | 4.272 | 256 | | | |
| | | | | | | | |
| | | | Parameter H | Estimates | | | |
| | | Par | ameter | Standar | rd | | |
| Variable | DF | Es | timate | Erro | or t | Value | Pr > t |
| Intercept | : 1 | 62 | .42741 | 5.582 | 18 | 11.18 | <.0001 |
| round1 | 1 | 0 | .13449 | 0.0764 | 41 | 1.76 | 0.0804 |

(a) (2 marks) Is there evidence of a linear relationship between golf scores on the first and second round of the tournament? Explain.

(b) (3 marks) The lowest score obtained on the first round was 66. Predict the second round score of the golfer who achieved this. Is this surprising? Explain your answer in terms of known facts about simple linear regression.

(Question 3 continued)

(c) (4 marks) Below are plots of the studentized residuals versus the predicted values, and a normal quantile plot of the residuals. What additional information do you learn from the plots? Be specific.



(Question 3 continued)

(d) (2 marks) In assignment 1 we considered the relationship between football kickers' field goal percentages one year with the percentage of field goals scored the previous year and found problems with violations of the Gauss-Markov conditions in the initial analysis. In the regression here we are examining the relationship between golf scores on one round with golf scores on the previous round. Do we have a similar problem with violations of the Gauss-Markov assumptions? Why or why not?

(e) (2 marks) The plot of the data below includes 95% confidence intervals for the mean score in round 2 given the score in round 1. About 90% of the data points fall outside the confidence limits. Explain how it can occur that so many observations are missed.

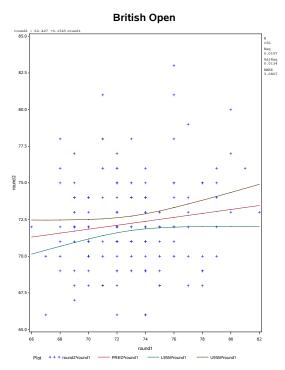


TABLE B.2 (concluded) Percentiles of the t Distribution.

| v .00 .70 .80 .50 .70 .80 .50 .51 1 0.233 0.727 1.376 1.961 3.078 6.314 1 0.233 0.727 1.376 1.961 3.078 6.314 1 0.233 0.559 0.617 1.061 1.365 3.078 6.314 1 0.235 0.559 0.920 1.1190 1.533 2.133 1 0.206 0.559 0.920 1.1190 1.476 2.015 1 0.206 0.543 0.895 1.1109 1.476 2.015 1 0.206 0.543 0.883 1.100 1.885 2.015 1 0.206 0.543 0.883 1.003 1.375 1.812 0.206 0.543 0.883 1.003 1.375 1.812 0.206 0.544 0.883 1.003 1.373 1.812 1 0.258 0.580 1.03 | 2.776 2.776 2.776 2.776 2.776 2.776 2.776 2.776 2.776 2.776 2.776 2.776 2.776 2.776 2.236 2.236 2.236 2.236 2.160 2.160 2.160 2.160 2.160 2.160 2.160 2.160 2.160 2.160 2.160 2.160 2.160 2.160 2.160 2.160 2.160 2.176 2.160 2.176 2.176 2.201 2.201 2.201 2.201 2.201 2.201 2.201 2.176 2.2010 | * | .985 21.205 3.5643 3.5643 3.5643 3.298 3.298 3.298 3.298 2.574 2.5775 2.57775 2.57775 2.57775 2.577755 2.57775556 2.5777556 2.57775566 2.57775566 2.57775666666666666666666666666666666666 | .99 31.821 6.965 4.546 3.365 3.365 3.143 3.143 3.143 3.143 3.143 2.744 2.764 2.764 2.764 2.764 2.764 2.764 | ,9925 42.434 8.073 5.047 5.047 5.047 5.047 5.048 3.634 3.634 3.634 3.372 3.375 | .995 63.657 9.925 9.925 9.664 4.032 4.032 3.707 | 5106. | 2666. 105 AFA |
|---|--|---|--|---|--|--|---------|------------------|
| 50 70 80 70 80 50 90 1 0.335 0.727 1.376 1.963 3.078 2 0.239 0.617 1.061 1.386 1.638 0.277 0.559 0.577 1.566 0.941 1.190 1.533 0.266 0.559 0.920 0.920 1.190 1.533 0.267 0.559 0.920 1.119 1.416 0.266 0.543 0.889 1.119 1.416 0.261 0.559 0.590 1.093 1.356 0.260 0.543 0.889 1.100 1.333 0.261 0.543 0.876 1.093 1.372 0.263 0.540 0.877 1.093 1.372 0.264 0.870 1.093 1.372 0.266 0.531 0.870 1.093 1.372 0.258 0.590 1.093 1.076 1.333 0.258 0.259 | 273, 2706, 2776, 2706, 2706, 472, 2003, 2776, 2776, 2776, 2776, 2776, 2776, 2776, 2776, 2776, 2776, 2716, 27 | | | 31.821 6.965 4.541 3.347 3.345 3.345 3.345 3.345 3.345 2.821 2.821 2.821 2.821 2.764 2.764 2.764 2.764 2.764 2.764 2.764 | 42404 8.073 5.047 5.047 5.047 3.058 3.053 3.055 3.055 3.055 | 63.657 63.657 9.925 9.925 4.604 4.032 4.032 3.707 | | 05 929 |
| 1 0.335 0.777 1.376 1.961 3.078 2 0.289 0.617 1.061 1.386 1.886 0.277 0.584 0.978 1.250 1.638 1.886 0.267 0.559 0.920 1.136 1.476 1.638 0.267 0.559 0.920 1.136 1.476 1.539 0.266 0.559 0.9906 1.119 1.476 0.263 0.549 0.8906 1.119 1.476 0.264 0.893 1.100 1.333 1.410 0.264 0.895 1.100 1.333 1.772 0.264 0.893 1.100 1.333 1.372 0.266 0.543 0.896 1.003 1.372 0.259 0.544 0.890 1.003 1.372 0.258 0.531 0.876 1.003 1.371 0.258 0.351 0.870 1.003 1.371 0.259 0.531 | 12,706 4,303 3,176 2,571 2,571 2,571 2,572 2,236 2,236 2,236 2,236 2,238 2,238 2,238 2,238 2,179 2,179 2,179 2,179 2,179 2,179 2,176 2,176 2,176 2,176 2,176 2,201 2,201 2,176 2,176 2,203 2,203 2,203 2,203 2,203 2,203 2,203 2,203 2,203 2,203 2,203 2,203 2,203 2,203 2,203 2,2776 2,2766 2,27 | | | 31.821 6.965 6.965 3.747 3.747 3.747 3.143 3.143 2.745 2.829 2.829 2.764 2.764 2.764 2.764 2.764 2.650 2.650 | 42.434 8.075 5.047 5.047 4.088 3.634 3.372 3.372 3.085 | 63.657 9.925 5.841 4.604 4.032 3.707 | | 626.50 |
| 0.289 0.617 1.061 1.361 1.365 1.476 1.476 1.476 1.476 1.475 1.475 1.475 1.475 1.475 1.475 1.475 1.475 1.475 1.476 <th< td=""><td>2,200 2,571 2,571 2,571 2,571 2,571 2,571 2,575 2,577 2,572</td><td>2223 222832 22345 2223 222832 2245 2223 222832 2245 2223 2228 2223 228 2223 228 2233 228 2234</td><td></td><td>6,9,05 3,747 3,143 3,143 2,143 2,143 2,143 2,143 2,262 2,262 2,262 2,262 2,262</td><td>8,005 5,047 3,634 3,534 3,372 3,372 3,085</td><td>5.841 5.841 4.604 4.032 3.707</td><td>127.322</td><td>20.00</td></th<> | 2,200 2,571 2,571 2,571 2,571 2,571 2,571 2,575 2,577 2,572 | 2223 222832 22345 2223 222832 2245 2223 222832 2245 2223 2228 2223 228 2223 228 2233 228 2234 | | 6,9,05 3,747 3,143 3,143 2,143 2,143 2,143 2,143 2,262 2,262 2,262 2,262 2,262 | 8,005 5,047 3,634 3,534 3,372 3,372 3,085 | 5.841 5.841 4.604 4.032 3.707 | 127.322 | 20.00 |
| 0.277 0.584 0.778 1.286 0.478 1.286 1.466 0.267 0.259 0.941 1.190 1.533 0.266 0.549 0.926 1.341 1.446 0.266 0.549 0.896 1.119 1.446 0.266 0.549 0.889 1.108 1.333 0.266 0.549 0.889 1.108 1.333 0.266 0.549 0.889 1.108 1.333 0.260 0.549 0.876 1.093 1.372 0.260 0.540 0.876 1.093 1.372 0.259 0.539 0.873 1.093 1.372 0.258 0.539 0.876 1.093 1.372 0.257 0.534 0.863 1.076 1.341 0.257 0.534 0.863 1.076 1.343 0.257 0.534 0.863 1.076 1.343 0.256 0.531 0.863 1.076 1.343 <td>4,303 2,571 2,571 2,571 2,571 2,571 2,571 2,571 2,571 2,572 2,572 2,572 2,572 2,572 2,572 2,572 2,572 2,572 2,5777 2,5777 2,5777 2,5777 2,57777 2,57777 2,5777777 2,57777777777</td> <td>2222823333244 22228233324 2222823325 2222823325 22228232325 22228232325 22228232325 22228232325 22228232325 22228232325 22228232325 22228232325 222282325 222282325 222282325 222282325 222282325 222282325 2222825 225 2255 25555 2555 25555 25555 25555 25555 25555 25555 25555 25555 25555 25555 25555 25555 255555 25555 255555 25555 255555 25555 25555 25555 25555 25</td> <td></td> <td>4.541 3.747 3.747 3.747 3.765 2.821 2.764 2.764 2.764 2.764 2.764 2.764 2.650 2.650</td> <td>5.047 4.088 3.574 3.372 3.085</td> <td>5.841 4.604 3.707</td> <td>14.089</td> <td>21.14</td> | 4,303 2,571 2,571 2,571 2,571 2,571 2,571 2,571 2,571 2,572 2,572 2,572 2,572 2,572 2,572 2,572 2,572 2,572 2,5777 2,5777 2,5777 2,5777 2,57777 2,57777 2,5777777 2,57777777777 | 2222823333244 22228233324 2222823325 2222823325 22228232325 22228232325 22228232325 22228232325 22228232325 22228232325 22228232325 22228232325 222282325 222282325 222282325 222282325 222282325 222282325 2222825 225 2255 25555 2555 25555 25555 25555 25555 25555 25555 25555 25555 25555 25555 25555 25555 255555 25555 255555 25555 255555 25555 25555 25555 25555 25 | | 4.541 3.747 3.747 3.747 3.765 2.821 2.764 2.764 2.764 2.764 2.764 2.764 2.650 2.650 | 5.047 4.088 3.574 3.372 3.085 | 5.841 4.604 3.707 | 14.089 | 21.14 |
| 0.271 0.559 0.971 1.550 0.971 1.568 0.267 0.559 0.926 1.136 1.440 0.261 0.559 0.926 1.136 1.440 0.261 0.559 0.926 1.136 1.440 0.261 0.549 0.889 1.100 1.533 0.261 0.549 0.889 1.100 1.415 0.260 0.540 0.875 1.008 1.347 0.260 0.540 0.875 1.003 1.333 0.259 0.537 0.870 1.003 1.345 0.258 0.537 0.870 1.003 1.333 0.257 0.531 0.865 1.071 1.333 0.257 0.533 0.865 1.071 1.333 0.256 0.531 0.865 1.071 1.333 0.257 0.533 0.865 1.071 1.333 0.256 0.531 0.866 1.076 1.333 | 2.182 2.571 2.571 2.571 2.576 2.576 2.576 2.576 2.576 2.187 2.187 2.187 2.187 2.187 2.187 2.187 2.187 2.187 2.187 2.187 2.187 2.187 2.187 2.187 2.187 2.182 2.182 2.298 2.191 | 4 2.39 6 2.39 6 2.33 10 2.33 11 2.33 12 2.25 12 2.2 | | 3.747 3.747 3.365 2.9143 2.988 2.898 2.764 2.764 2.718 2.768 2.768 2.681 2.681 2.681 | 4,083 3,634 3,372 3,203 3,085 | 4.604 4.032 3.707 | 7.453 | 12.92 |
| 0.267 0.359 0.941 1.190 1.533 0.266 0.559 0.920 1.136 1.476 0.266 0.559 0.920 1.136 1.440 0.266 0.549 0.896 1.119 1.410 0.261 0.549 0.896 1.119 1.440 0.261 0.549 0.895 1.100 1.333 0.262 0.549 0.883 1.100 1.383 0.260 0.549 0.876 1.093 1.372 0.259 0.539 0.876 1.093 1.372 0.259 0.539 0.876 1.093 1.372 0.258 0.531 0.860 1.076 1.333 0.257 0.531 0.860 1.076 1.333 0.256 0.531 0.860 1.066 1.333 0.256 0.531 0.860 1.066 1.333 0.256 0.531 0.860 1.066 1.333 0.256 | 2.776 2.571 2.571 2.571 2.576 2.365 2.238 2.238 2.238 2.238 2.145 2.145 2.145 2.145 2.145 | 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 | | 3.365 3.143 2.988 2.821 2.764 2.718 2.681 2.681 2.658 | 3.634 3.372 3.203 3.085 | 4.032 3.707 | 5.598 | 8.61 |
| 0.256 0.533 0.920 1.136 1.440 0.265 0.534 0.885 1.119 1.440 0.265 0.546 0.885 1.110 1.415 0.265 0.546 0.885 1.100 1.415 0.261 0.545 0.885 1.100 1.333 0.260 0.546 0.883 1.100 1.333 0.260 0.540 0.876 1.003 1.372 0.250 0.530 0.876 1.003 1.372 0.259 0.537 0.870 1.003 1.373 0.258 0.534 0.865 1.076 1.341 0.257 0.534 0.865 1.076 1.343 0.257 0.534 0.865 1.076 1.343 0.257 0.534 0.865 1.076 1.333 0.256 0.531 0.561 1.066 1.333 0.256 0.531 0.561 1.066 1.333 0.256 | 2.571 2.447 2.365 2.306 2.306 2.201 2.201 2.179 2.145 2.145 2.145 2.145 | 6 2.50 9 2.24 10 2.23 10 2.23 | | 3.143 2.998 2.896 2.821 2.821 2.718 2.631 2.650 2.624 | 3.372 3.203 3.085 | 3.707 | 4.773 | 6.86 |
| 0.256 0.553 0.906 1.134 1.440 0.261 0.549 0.889 1.119 1.415 0.261 0.549 0.889 1.108 1.397 0.261 0.543 0.889 1.108 1.397 0.261 0.543 0.889 1.108 1.397 0.260 0.543 0.876 1.093 1.372 0.259 0.539 0.876 1.093 1.372 0.259 0.537 0.876 1.079 1.372 0.259 0.537 0.876 1.079 1.373 0.258 0.534 0.863 1.076 1.345 0.257 0.534 0.863 1.076 1.345 0.257 0.534 0.863 1.076 1.345 0.257 0.534 0.863 1.076 1.333 0.256 0.531 0.863 1.076 1.333 0.256 0.531 0.853 1.069 1.333 0.2566 | 2,447 2,365 2,366 2,366 2,365 2,268 2,129 2,145 2,145 2,146 2,146 2,146 | 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | | 2.529 2.896 2.821 2.718 2.718 2.650 2.624 | 3.203 | June . | C13 P | S 0.6 |
| 7 0.263 0.549 0.896 1.119 1.415 8 0.262 0.546 0.889 1.108 1.397 9 0.266 0.546 0.889 1.100 1.415 9 0.266 0.549 0.889 1.100 1.383 9 0.266 0.549 0.876 1.093 1.372 9 0.269 0.539 0.876 1.093 1.373 9 0.259 0.539 0.877 1.093 1.356 9 0.258 0.537 0.870 1.079 1.341 9 0.258 0.536 0.865 1.076 1.345 0.257 0.534 0.863 1.076 1.333 0.257 0.531 0.861 1.069 1.333 0.256 0.531 0.861 1.066 1.333 0.256 0.531 0.861 1.063 1.333 0.256 0.531 0.863 1.063 1.333 <td>2,236 2,246 2,246</td> <td>222 222 222 222 222 222 222 222 222 22</td> <td></td> <td>2.524 2.754 2.754 2.650 2.624 2.624</td> <td>3,085</td> <td>2 400</td> <td>000 1</td> <td>14.4</td> | 2,236 2,246 | 222 222 222 222 222 222 222 222 222 22 | | 2.524 2.754 2.754 2.650 2.624 2.624 | 3,085 | 2 400 | 000 1 | 14.4 |
| 8 0.262 0.546 0.889 1.108 1.979 0.261 0.543 0.883 1.100 1.372 0.260 0.543 0.883 1.100 1.372 0.260 0.543 0.875 1.003 1.372 0.250 0.543 0.876 1.083 1.372 0.259 0.537 0.876 1.083 1.356 0.259 0.537 0.876 1.075 1.333 0.258 0.537 0.866 1.076 1.333 0.257 0.534 0.865 1.076 1.333 0.257 0.534 0.865 1.076 1.333 0.256 0.531 0.861 1.066 1.333 0.256 0.531 0.861 1.066 1.333 0.256 0.531 0.861 1.066 1.325 0.256 0.531 0.561 1.063 1.323 0.256 0.531 0.856 1.063 1.323 | 2,230 2,230 2,230 2,230 2,230 2,130 | 2223 2228 2228 2228 2228 2228 2228 2228 | | 2.83% 2.764 2.718 2.718 2.650 2.650 | 0000 | 1000 | 670.4 | # 8 4 4 |
| 0.261 0.543 0.883 1.100 1.397 0.260 0.543 0.883 1.100 1.372 0.260 0.540 0.875 1.003 1.372 0.259 0.539 0.876 1.083 1.363 0.259 0.537 0.876 1.083 1.363 0.259 0.537 0.870 1.079 1.363 0.258 0.537 0.866 1.074 1.345 0.257 0.354 0.863 1.076 1.345 0.257 0.354 0.863 1.071 1.317 0.257 0.534 0.863 1.076 1.333 0.257 0.534 0.863 1.076 1.333 0.257 0.531 0.863 1.066 1.333 0.256 0.531 0.863 1.066 1.333 0.256 0.531 0.856 1.066 1.333 0.256 0.531 0.856 1.066 1.333 0.256 | 2,2306 2,232 2,223 2,220 2,200 2,220 2,200 | 9 2339 10 2339 11 2332 13 2228 14 2228 15 2228 16 2228 17 22 17 22 1 | | 2.821 2.764 2.718 2.624 2.624 | N CONTRACTOR | 5.300 | 3.835 | さら |
| 0.256 0.354 0.875 1.100 1.383 0.260 0.549 0.876 1.093 1.372 0.259 0.539 0.876 1.093 1.373 0.259 0.539 0.876 1.093 1.363 0.259 0.537 0.876 1.073 1.363 0.259 0.537 0.876 1.073 1.356 0.258 0.536 0.865 1.074 1.341 0.257 0.534 0.865 1.076 1.333 0.257 0.534 0.863 1.076 1.333 0.257 0.534 0.861 1.069 1.333 0.256 0.531 0.861 1.063 1.333 0.256 0.531 0.861 1.063 1.333 0.256 0.531 0.861 1.063 1.333 0.256 0.531 0.853 1.063 1.333 0.256 0.531 0.853 1.063 1.312 0.256 | 2,232 2,228 2,201 2,110, | 10 2.33 13 2.25 13 2.25 15 2.25 16 2.25 17 2.2 | | 2.764 2.718 2.630 2.624 | 2.998 | 3.250 | 3.690 | 4,78 |
| 0.250 0.340 0.870 1.093 1.372 0.250 0.340 0.876 1.083 1.366 0.259 0.337 0.870 1.083 1.356 0.258 0.337 0.876 1.083 1.356 0.258 0.337 0.866 1.076 1.385 0.258 0.335 0.866 1.076 1.345 0.257 0.354 0.865 1.071 1.345 0.257 0.334 0.865 1.071 1.345 0.257 0.534 0.862 1.067 1.333 0.256 0.531 0.861 1.066 1.333 0.256 0.531 0.861 1.066 1.325 0.256 0.531 0.856 1.063 1.323 0.256 0.531 0.856 1.063 1.323 0.256 0.531 0.856 1.063 1.312 0.256 0.531 0.856 1.063 1.312 0.256 | 2,228 2,179 2,176 2,145 2,145 2,145 | 11 2.33 13 2.28 13 2.28 14 2.28 15 2.28 16 2.28 17 2.28 17 2.28 | | 2.718 2.681 2.650 2.624 | 2.932 | 3.169 | 3.581 | 4.58 |
| 0.256 0.340 0.876 1.08 1.363 0.229 0.339 0.873 1.033 1.356 0.229 0.337 0.873 1.033 1.356 0.229 0.337 0.873 1.073 1.356 0.228 0.357 0.866 1.076 1.345 0.257 0.354 0.865 1.076 1.345 0.257 0.534 0.863 1.076 1.341 0.257 0.534 0.863 1.076 1.343 0.257 0.534 0.863 1.069 1.333 0.257 0.533 0.863 1.066 1.333 0.256 0.533 0.860 1.066 1.333 0.256 0.533 0.863 1.063 1.333 0.256 0.533 0.856 1.063 1.333 0.256 0.533 0.856 1.063 1.333 0.256 0.533 0.856 1.063 1.333 0.256 | 2.201 2.179 2.160 2.145 2.131 | | | 2.650 2.650 2.624 | 000 | 2 100 | COF 6 | 1 |
| 0.259 0.539 0.873 1.083 1.356 0.1259 0.537 0.870 1.079 1.356 0.1258 0.537 0.866 1.076 1.345 0.1258 0.537 0.866 1.076 1.345 0.1258 0.535 0.866 1.074 1.341 0.1257 0.534 0.865 1.074 1.341 0.1257 0.534 0.865 1.076 1.333 0.1257 0.534 0.865 1.069 1.333 0.1257 0.534 0.860 1.066 1.333 0.1257 0.534 0.860 1.066 1.333 0.1256 0.531 0.860 1.066 1.333 0.1256 0.531 0.856 1.066 1.323 0.256 0.531 0.856 1.063 1.318 0.256 0.531 0.855 1.063 1.318 0.256 0.531 0.856 1.063 1.318 0. | 2113 2116 2115 2115 2115 2115 | | | 2.624 | 2007 | 001.0 | 144.0 | ÷÷ |
| 0.259 0.337 0.870 1.000 1.000 1.000 0.258 0.337 0.866 1.076 1.345 0.258 0.336 0.866 1.076 1.345 0.257 0.836 1.076 1.345 0.257 0.534 0.865 1.076 1.345 0.257 0.534 0.865 1.071 1.337 0.257 0.534 0.862 1.069 1.333 0.257 0.533 0.862 1.066 1.333 0.257 0.533 0.860 1.066 1.333 0.257 0.533 0.860 1.066 1.325 0.256 0.531 0.850 1.066 1.323 0.256 0.531 0.853 1.063 1.319 0.256 0.531 0.856 1.063 1.318 0.256 0.531 0.856 1.063 1.318 0.256 0.531 0.856 1.063 1.318 0.256 | 2.179 2.145 2.131 2.131 | 12 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 | | 2.624 | 2.330 | CCU.C | 274-5 | 1.4 |
| 0.228 0.337 0.368 1.079 1.350 0.258 0.336 0.366 1.076 1.345 0.257 0.356 0.865 1.076 1.345 0.257 0.534 0.865 1.076 1.345 0.257 0.534 0.865 1.076 1.345 0.257 0.534 0.863 1.076 1.333 0.257 0.534 0.863 1.069 1.333 0.257 0.533 0.863 1.066 1.333 0.257 0.533 0.863 1.066 1.333 0.256 0.533 0.853 1.063 1.325 0.256 0.533 0.856 1.063 1.323 0.256 0.531 0.856 1.063 1.323 0.256 0.531 0.856 1.063 1.323 0.256 0.531 0.856 1.063 1.316 0.256 0.531 0.856 1.063 1.316 0.256 | 2160 2145 2131 | 11 228 15 228 16 2233 2335 2356 2357 2357 2357 2357 2357 2357 2357 2357 | | 2.624 | 2.801 | 3.012 | 3.372 | 4.2 |
| 0.258 0.336 0.466 1.076 1.345 0.258 0.335 0.866 1.074 1.341 0.257 0.534 0.865 1.074 1.341 0.257 0.534 0.865 1.074 1.341 0.257 0.534 0.865 1.069 1.333 0.257 0.534 0.863 1.069 1.333 0.257 0.534 0.863 1.066 1.333 0.257 0.533 0.860 1.066 1.333 0.256 0.533 0.858 1.061 1.323 0.256 0.531 0.853 1.061 1.319 0.256 0.531 0.856 1.063 1.318 0.256 0.531 0.856 1.063 1.318 0.256 0.531 0.856 1.053 1.316 0.256 0.531 0.855 1.053 1.316 0.256 0.531 0.855 1.053 1.316 0.256 | 2.145 | 15 2.24 16 2.23 17 2.23 | | | 2.771 | 2.977 | 3.326 | 4.15 |
| 0.256 0.531 0.860 1.014 1.341 0.257 0.253 0.865 1.001 1.333 0.257 0.254 0.865 1.001 1.333 0.257 0.534 0.865 1.007 1.333 0.257 0.534 0.865 1.065 1.333 0.257 0.533 0.860 1.065 1.333 0.257 0.533 0.860 1.065 1.333 0.256 0.531 0.850 1.061 1.325 0.256 0.531 0.853 1.061 1.321 0.256 0.531 0.857 1.063 1.319 0.256 0.531 0.856 1.063 1.318 0.256 0.531 0.856 1.053 1.318 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.855 1.058 1.316 0.256 0.531 0.855 1.057 1.316 | 2.131 | 10 2.23 | | 2.602 | 2.746 | 2.947 | 3.286 | 4.93 |
| 0.258 0.535 0.865 1.071 1.337 0.257 0.257 0.534 0.863 1.069 1.333 0.257 0.257 0.533 0.863 1.066 1.333 0.257 0.533 0.863 1.066 1.333 0.257 0.533 0.861 1.066 1.333 0.257 0.533 0.861 1.066 1.325 0.256 0.531 0.858 1.061 1.325 0.256 0.531 0.857 1.063 1.319 0.256 0.531 0.856 1.063 1.318 0.256 0.531 0.856 1.063 1.318 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.855 1.058 1.316 0.256 0.531 0.855 1.058 1.316 0.256 0.531 0.855 1.057 1.316 <td></td> <td>17 2.23</td> <td></td> <td>1.92 0</td> <td>PCL C</td> <td>100 0</td> <td>026 E</td> <td>40 W</td> | | 17 2.23 | | 1.92 0 | PCL C | 100 0 | 026 E | 40 W |
| 0.257 0.254 0.863 1.069 1.333 0.257 0.234 0.863 1.067 1.333 0.257 0.333 0.861 1.067 1.333 0.257 0.333 0.861 1.066 1.335 0.257 0.533 0.860 1.061 1.325 0.256 0.532 0.858 1.063 1.325 0.256 0.532 0.858 1.061 1.321 0.256 0.531 0.856 1.063 1.319 0.256 0.531 0.856 1.063 1.316 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.855 1.057 1.316 | - UCL C | 2224 C | | 195 6 | 201.0 | NON C | CCC 1 | 100 |
| 0.257 0.234 0.862 1.067 1.330 0.257 0.333 0.860 1.067 1.330 0.257 0.333 0.860 1.066 1.328 0.257 0.533 0.860 1.066 1.328 0.257 0.533 0.860 1.066 1.328 0.256 0.532 0.858 1.061 1.321 0.256 0.531 0.858 1.060 1.319 0.256 0.531 0.857 1.060 1.319 0.256 0.531 0.856 1.060 1.319 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.855 1.058 1.316 0.256 0.531 0.855 1.057 1.314 | 011 0 | 10 0 51 | | 03 6 | 089 6 | 279 270 | 201 1 | 8 |
| 0.257 0.333 0.361 1.066 1.328 0.257 0.333 0.360 1.066 1.328 0.257 0.533 0.360 1.064 1.325 0.256 0.532 0.858 1.061 1.323 0.256 0.532 0.858 1.061 1.323 0.256 0.531 0.857 1.061 1.319 0.256 0.531 0.857 1.063 1.319 0.256 0.531 0.857 1.063 1.316 0.256 0.531 0.856 1.038 1.316 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.855 1.058 1.316 0.256 0.531 0.855 1.057 1.316 | 101 6 | xis. | | 2003 6 | 100-1 | 130 0 | | 100 5 |
| 0.257 0.533 0.860 1.064 1.325 0.256 0.532 0.859 1.064 1.325 0.256 0.532 0.858 1.061 1.321 0.256 0.532 0.858 1.061 1.321 0.256 0.531 0.856 1.063 1.316 0.256 0.531 0.856 1.063 1.318 0.256 0.531 0.856 1.063 1.316 0.256 0.531 0.856 1.053 1.316 0.256 0.531 0.856 1.053 1.316 0.256 0.531 0.855 1.053 1.316 0.256 0.531 0.855 1.057 1.316 | 10117 | 00 - F | | 000 0 | 10.4 | 100.2 | | |
| 0.257 0.532 0.859 1.063 1.323 0.256 0.532 0.858 1.061 1.321 0.256 0.532 0.858 1.061 1.321 0.256 0.532 0.858 1.061 1.321 0.256 0.531 0.857 1.065 1.318 0.256 0.531 0.857 1.059 1.318 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.855 1.058 1.315 | 560-7 | | | 070-7 | 10077 | C+0-7 | CC1.C | 5 |
| 0.257 0.332 0.889 1.063 1.323 0.256 0.532 0.858 1.061 1.321 0.256 0.532 0.858 1.060 1.319 0.256 0.531 0.857 1.060 1.319 0.256 0.531 0.857 1.063 1.319 0.256 0.531 0.856 1.053 1.316 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.855 1.057 1.314 | 02017 | 21 2.18 | | 2.518 | 2.649 | 2.831 | 3.135 | 3.81 |
| 0.256 0.531 0.858 1.061 1.321 0.256 0.531 0.858 1.060 1.319 0.256 0.531 0.856 1.039 1.316 0.256 0.531 0.856 1.038 1.316 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.855 1.057 1.314 | 2.080 | 22 2.18 | | 2.508 | 2.639 | 2.819 | 3,119 | 31.20 |
| 0.256 0.531 0.858 1.060 1.319 0.256 0.531 0.857 1.059 1.318 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.856 1.058 1.315 0.256 0.531 0.855 1.058 1.315 | 2.074 | 23 2.17 | | 2.500 | 2.629 | 2.807 | 31,004 | 3.7 |
| 0.256 0.531 0.857 1.058 0.256 0.531 0.857 1.058 1.058 0.51 0.556 0.51 0.556 1.058 1.058 0.256 0.51 0.855 1.055 0.51 0.855 1.057 1.151 | 2.069 | 74 217 | | C07 C | 2 600 | LIGT C | 160.5 | E F |
| 0.256 0.531 0.856 1.058 1.316 0.256 0.531 0.856 1.058 1.315 0.256 0.531 0.855 1.057 1.314 | 2.064 | 25 2.16 | | 2 485 | 2 612 | 787.0 | 31.078 | 15 |
| 0.256 0.531 0.856 1.058 1.315 0.256 0.531 0.855 1.057 1.314 | 2.060 | 31 5 36 | | | 209 0 | OFF C | 2,067 | CURL R. |
| 0.256 0.531 0.855 1.057 1.314 | 750 6 | 01.4 02 | | | 000 6 | 100 0 | 1000 | 10 |
| | 00012 | | | | 0607 | 1111 | 1000 | 3 4 5 e |
| 0.256 0.530 0.855 1.055 | 2:052 | 28 2.15 | | 2.407 | 2.592 | 2,703 | 140.5 | 0.0 |
| 0.256 0.570 0.664 0.570 | 2.048 | 29 2.15 | | 2.462 | 2.586 | 2.756 | 3.038 | 3.6 |
| | 2.045 | 30 2.14 | | 2.457 | 2.581 | 2.750 | 3.030 | 3.6 |
| 0.834 1.055 1.310 | 2.042 | - | | FCA F | CP3 C | SUL C | 100 % | 5 6 |
| . 1.050 1.303 | 1001 | -11-7 01-4 00-6 | ł | 002 2 | TUS C | 2 660 | 2014 | ST E |
| 0.254 0.527 0.848 1.045 1.296 | 1000 | 1000 | | 936 6 | 031 6 | 20011 | 19860 | i r |
| 0.526 0.845 1.041 1.780 | 1 000 | 38 | | 900.4 | 001-7 | 11013 | 20012 | 10 |
| 0.524 0.842 1.036 1.262 | 1.900 | | | 0707 | 70677 | 0/07 | 1100.7 | 4.0 |

Entry is $t(A; \nu)$ where $P\{t(\nu) \le t(A; \nu)\} = A$ TABLE B.2 Percentiles of the *t* Distribution.

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